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# Real and Presumed Categories: A Formal Approach[1]

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**ABSTRACT:** Categorization is fundamental to any scientific pursuit, posing the question of what categories themselves amount to. Are categories absolute or mere linguistic constructs? If the latter, are they to be trusted? Otherwise, what can one use to ground categories? The present piece reminds us that one can go about this substantively or formally. Substantive, for beings with a neuro-physiological nature, boils down to how we happen to be "wired" or how our brain "works", whatever that means. Formal ought to be grounded on some abstract system, of the sort that logic and arithmetic represent — given "the unreasonable effectiveness of mathematics". In this context, I consider whether primitive linguistic (distinctive) features are substantive or formal. In the end, I suggest that this depends on the nature of the feature, and that in some fundamental sense there is ample space for foundational formal features, which has a variety of technical and philosophical consequences worth pursuing.

KEYWORDS: categorization; formal vs. substantive; distinctive features; linear algebra; matrix syntax; minimalism.

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[1] This piece is to celebrate the life and career of José Ignacio Hualde, may it continue for decades to come. I wish to acknowledge Naomi Feldman and the participants in our 2022 seminar at UMD for helpful comments, as well as their roles in developing these general ideas to the following colleagues: Michael Jarret, Román Orús, Ángel Gallego, and especially the late Roger Martin. My appreciation goes also, for various reasons, to Bill Idsardi, Diego Krivochen, Dan Lathrop, Steve Marcus, Dave Medeiros, Massimo Piattelli-Palmarini, Doug Saddy, Zach Stone, and Howard Lasnik, who discussed the foundations of some of these proposals as they appear in Lasnik & Uriagereka (2022). All errors, misinterpretations, or speculations are my responsibility.

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## 1. Categorial Problems

How can one pursue any scientific study without presuming categories to support it? Then again, the moment one categorizes, is one not *imposing* a particular bias on one's subject matter—thereby proceeding to reap just what was sowed? Since antiquity, the greatest philosophers, from Aristotle to Kant, devoted decades of study and pages too numerous to review to this matter. The brave Stagirite famously isolated 10 primitive categories, which made the man from Königsberg balk at the lack of elegance. Scholastic philosophers subdivided the classical categories into two sets, of four primaries (of *substance*, *relation*, *quantity*, and *quality*) and six secondaries (of *place*, *time*, *situation*, *condition*, *action*, and *passion*). By the 18th century (culminating in Descartes's dualism) Kant cut to the chase by reducing substantive claims to relational analyses, whence *disjunction*, *causality*, and *inherence*, mixing logic, physics, and even some metaphysics for the task (this, without going into secondary categories, also dabbing on *community* and similar sociological ideas, as well as more airy notions like *will*...) It is not surprising that by the time this all reaches Wittgenstein, through Russell and Whitehead, the entire shebang turns into a linguistic exercise, seen as a game of some kind, even a metaphor (in due time), as it all gets postmodern. So where does all that leave science?

Working scientists never cared much about what philosophers said regarding their concepts, thus marching on to postulate bizarre interactions (gravity, a probability wave...) and of course the categories those happen to hold of (mass/energy, sub-atomic particles...). No physicist cares about philosophers insisting on the putative irrationality of entanglement ("spooky action at a distance"), if experiments show it to be real, with unbelievable predictability. The problem is not trivial for, with scientific analyses, even the cherished causality goes, at least as Kant would have understood it—which is what gave Einstein the creeps. Then again, "it is what it is", and if the modern synthesis has taught us something, this is to trust more the scientist than the philosopher, at least when it comes to putting a tin into outer space or addressing a pandemic. To be fair: what carried the best scientists to their conclusions, aside of course from the ingenuity of great experimental results, is mathematics, without which their progress would have been harder.

It must be noted: some philosophers/logicians, like Pierce most notably in modern times, did invoke the idea of the relations between propositions as being spatial, in a rigorous sense, as is also seen (with equal rigor at the relevant level of abstraction) in the interactions of language, where we have verbal "valence" linking predicates to subjects in linguistically familiar ways. While there could be something scary to all of this if one takes human languages to be cultural games, things take a different turn if the faculty of language reflects some deep natural law (just like physics does, in a Galilean sense). This of course is the Chomskyan view (Chomsky 1955, 1965, 1981, 1995), although Chomsky himself notably attributed it (in different ways, see e.g. Chomsky 1972) to Humboldt and other luminaries. Attribution aside though, one can imagine a neo-Peircean approach where the predicates of language are organized in Neo-Davidsonian fashion—reflecting "theta roles" building on *themes*, possibly introducing *goals*, eventually capping it all off in *causes*, see Hale & Keyser (2002)—with category-inducing dependencies reflecting on a natural scaffolding (and secondary predications added to the mix, as relevant). This, to be sure, is all linguistic, but not because of that any less foundational than it would be to call it (meta-)physics.

One of the most important tools ever postulated by linguists are "distinctive features". The idea was around since antiquity, but it was revamped by Structuralists, as systematized in Jakobson *et al.* (1952) and Jakobson & Halle (1971), the culmination of which is Noam Chomsky and Morris Halle's foundational *Sound Pattern of English* (1968), which (owing much to Harris 1944) goes well beyond taxonomies, into nuanced computational processes that are still being debated (particularly within formal syntax). Here I want to take another stab at the distinction between categories and interactions thereof, in terms of an algebraic interpretation of distinctive features, which in a sense takes us back to old Peircian matters, but from a slightly different formal place, based on matrix mechanics. Examples of features include ±*sonorant* in phonology, ±*plural* in morphology, ±*telic* in semantics, ±N in syntax, for each of which, if they do not subdivide further, enough of a categorial status is presumed not to seek deeper explanations for their existence within our field of study. This presupposes levels-of-representation (phonology, morphology, semantics, syntax, or whatever) which bundle the features in point in different ways, according to relevant "paradigmatic geometries" and "syntagmatic dependencies"—presuming, in that, further categorial questions as to what those levels *are* (categories?) and how they relate (interactions?).

There seem to be two approaches to how to ground these general ideas: substantive and formal. (I am aware of the irony of introducing this further categorization, but I return to it.) Let's consider each in turn, as abstractly as needed, and since color is a classic in these domains, let's start by asking about the reality of features, first, from that perspective. Despite much ink in this regard, the experience of color appears to have relatively little to do with classical "color theory" (the "color wheel" and all that) or the "psychological effect" of colors, legendarily pursued by van Gogh and, later, the abstract expressionists. More mundanely, color perception correlates with the activation of light-sensitive (retinal) cones, the types of which allow for various nuances (selectively deactivated in "color-blindness"), which in some species permit perception beyond the human visible spectrum (see Palmer 1999). That mental event can be activated in the absence of light stimuli, say in dreams at night, but that in itself says (relatively) little more than there are neuro-physiological correlates of the cone-dependent trigger, which may be (more or less hallucinatorily) induced as a physical event of its own, whatever its cause. My point is simply that all of this is pretty substantive: change the relevant brain conditions and you get color blindness; there is nothing formal about the discussion, just an amazing natural phenomenon.

To that substantive approach, one may want to compare a formal one, as presumed for the distinction between 1 and –1. Although of course one can perceive instances of this (e.g., having vs. owing a beer), there is no particular relation between number one and *having*, let alone *beers*. One could express the exact same abstract idea with a temperature being above or below freezing or a date happening before or after noon. In short, the notion is algebraic, and it has innumerable formal consequences—even for the featural system we are now considering, where it makes sense to ask whether a plus value in an attribute means *presenting* as opposed to *lacking* it

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(*or*, in a different implementation, presenting the attribute in an *upward entailing* v. *downward entailing* scale). While it is sensible to have found light-sensitive cones reacting to different radiation frequencies, across animal kingdoms, it would be more surprising if it turned out that we have "plus sensitive" neurons or "minus activated" neuronal networks. Moreover, it is not obvious that an active control of these algebraic nuances is present in other animals, even closely related primates.

I hasten to add that no one has a clue what it means for us to interpret mathematics, just as no one understands why nature seems to "speak its language" (as Eugene Wigner called it in 1960, "the unreasonable effectiveness of mathematics"). But our most foundational sciences crucially depend on math to make their extraordinarily unintuitive claims about the categories of the universe, and it would seem as if, numerically inclined as (some) other animals might be, to some extent, no other species suspends its common-sensical disbelief about reality on the basis of scientific arguments. Some hate math and resist its conclusions, not wanting to get vaccinated and all that; it is easier to believe whatever "comes natural" in a common-sensical way. But a good number of us are willing to be persuaded by the reasoning and correlating evidence, which we think of as *natural* philosophy or, in a word, *science*. My only point now is that the features that depend on the algebraic scaffolding of science are *formal* not substantive, in the sense of cones or any other neurophysiological (as the case may be, genetic, proteomic, micro-biomic...) reality.

The question I want to consider now is whether primitive linguistic features are substantive (like color perception) or formal (like the understanding of ±1). Disappointingly, my answer is going to be that "it depends"—though I will focus on the formal type.

## 2. Syntactic Categories

A substantive approach to linguistic features is to liken them to "vision maps", mediated by brain physiology. This approach is promising, for instance, for *voice onset time* (VOT), associated by Poeppel (2003) to specific "brain events" we need not discuss now. To be sure, even in phonetics one has more at stake than VOT (place and manner of articulation, ballistic vs. sustained gestures, coordination among segments, the whole nine yards). None of that is particularly mystical, and the task ahead is to literally dissect and then integrate all that is relevant into some model. At the end of the day, we should come up with a sort of mental map of the sort my colleague Bill Idsardi is pursuing for phonemes, perhaps with some bits of the ensemble being more "hard wired" than others (though I have no ax to grind on whether much of this, in this particular substantive sense, comes from nature or is also acquired from nurture). It may take a generation, a century, or a millennium to complete this project, but it seems every bit as reasonable as it has been to determine the shape of proteins.

But I would like to focus here on a different—and I emphasize: entirely compatible—approach to feature dimensions: that their nature is algebraic and, hence, in a sense *prior* to the system's substance. So that we get this out of the way, it is reasonable to presume that the priority in point is actually physical, in a literal sense (following from the underlying biophysics of language); but it is fine for the point I need to make if the formal nature of the presumed dimensions stems from something deeper.

I also have no ax to grind on that ultimately philosophical matter. What concerns me now is a simple intuition that cuts across all known "systems" of linguistic features (holding of different levels of representation): the fact that they reflect realities that, in some fundamental sense, are "punctual" vs. "distributed". The easiest manifestation of this, phenomenologically, is saying "aaaaaaah" (as one does at the doctor) vs. illustrating the plosiveness of a "p", a "t", or a "k" for foreign students of a language like English. Plainly, "aaaaaaah" takes a second or two to produce (or the doctor will ask you to open your mouth again), while something like a "p" seems instantaneous (of course nothing is, but it *feels* that way). One hopes that the brain event correlating to each of these phenomena is different; but for now what interests me is only the distinction itself, which is present in various aspects of language. It seems best to think of that distinction in formal (not substantive) terms.

I will make my argument in a roundabout fashion, starting with a proposal I believe is one of the deepest in the history of linguists. In its modern instantiation, we can find it in the unpublished *Amherst Lectures* that Chomsky gave in 1974, when he was establishing the bases of what was later to be called the Principles & Parameters model associated to the "biolinguistcs" enterprise. Varro had a similar intuition in *De Lingua Latina* (Vol 2: Book IX, XXIV-31), when reminding us how "… the Greeks have divided speech into four parts, one in which the words have cases, a second in which they have indications of time, a third in which they have neither, a fourth in which they have both" [translated by Roland G. Kent in 1938]. The following is Chomsky's specific categorization (1974: lecture 3, p. 2):

> As far as the categorial component is concerned, it seems to me plausible to suggest that it is a kind of projection from basic lexical features through a certain system of schemata as roughly indicated in [1] and [2]:
>
> (1) [±N, ±V]: [+N, –V] = N[oun]; [+N, +V] = A[djective]; [–N, +V] = V[erb], [–N, –V] = everything else;
>
> (2) $X^n \rightarrow \ldots X^{n-1} \ldots$, where $x^i = [a = \pm N\ , b = \pm V\ ]^i$ and $X^1 = X$.
>
> Let us assume that there are two basic lexical features N and V (±N, ±V). Where the language has rules that refer to the categories nouns and adjectives. . . they will be framed in terms of the feature +N and where there are rules that apply to the category verbs and adjectives, . . . in terms of the feature +V.

When reflecting on these issues, the first thing to bear in mind is that there are four types of syntactic categories these authors are attempting to understand: nouns, verbs, adjectives, and "elsewhere". Instead of just listing these four, they pursue the intuition that each of these share commonalities, two-at-a-time, which leads them to postulate two abstract features with positive and negative values. Pursuing a traditional intuition going back to Greek philosophy, if not before, Chomsky assumes "that there are two lexical features N and V" (not A, etc.), nominal and verbal dimensions being more essential. Chomsky (1974: lecture 3, p. 3) also asserts that basic phrase-markers are "projected from the lexical categories uniformally", for "in a fundamental way the expansion of major categories like NP, VP, AP is independent of categorial choice of the head . . . [as] instantiations of the same general schemata." This of course are the origins of X'-theory, which underlies the minimalist *bare phrase structure*.

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The only footnote I have added to this important discussion is that those intuitions with a substantive tinge to them can be made sense of formally, if the implicit "conceptual orthogonality" is to be precisely stated through mathematical orthogonality:

(3) *Fundamental Assumption*
The V dimension is a (mathematical) transformation over an orthogonal N dimension.

Instantiating (3) in the complex plane, we can conclude:

(4) *Fundamental Corollary*
The N dimension has unit value 1 and the V dimension, unit value $i$; $[\pm N, \pm V] = [\pm 1, \pm i]$.

The Fundamental Corollary has the added advantage that it allows for the algebraic operation with Chomsky's feature matrices. In his 1974 piece, Chomsky was also interested in "subsidiary features" relating to "higher order endocentric categories", of the sort we call "grammatical" or "functional". Orús *et al.* (2017) pursues that program rigorously by placing the numerical values in (4) in the diagonal of the simplest (2×2) diagonal matrices. The result is a group for matrix multiplication that generalizes into a vector (Hilbert) space of the sort in Smolensky & Legendre (2006) (see also Smolensky 1990), albeit starting from purely syntactic assumptions. (The idea that mathematical knowledge for humans is directly related to language goes back to Alfred Wallace, see Eiseley 1961.) Note that Chomsky *ordered* his features in the familiar fashion [±N, ±V], which may be different from expressing the mere *presence* of ±N and ±V features—if the brackets imply order, which is obviously different from [±V, ±N]. Orús *et al.* (2017), presuming (3)/(4), then argue for:

(5) a. $\begin{bmatrix} \pm 1 & 0 \\ 0 & \pm i \end{bmatrix}$ b. $\begin{bmatrix} \pm i & 0 \\ 0 & \pm 1 \end{bmatrix}$

These can be treated as linear operators with standard properties (matrix traces and determinants, characteristic polynomials, eigenvalues, and so on).

In the remainder of this note, I will argue that both of these types of linguistic features (substantive and formal) make good sense for the language faculty to have, but only the latter constitute a solid base for a notion of "categoricity" that can be soundly defended, with consequences for not just how to separate linguistic categories from their interactions, but also what could their "mental representation" be, if that is the right way to speak about such matters in neurophysiological terms. Let us tackle each of these ideas in turn.

## 3. Operating with Matrices

I will not dwell on this too much now, but just give the gist of the proposal, and bearing in mind that multiplication is a scaling operation. The project I have just alluded to basically explores, as a foundation, the relation between a lexical item and whatever it combines with, in terms of a relation we may call Merge, beginning with the four Chomsky lexical categories that, in this particular guise, are the 2×2 matrices in 6:

$$\begin{array}{lllll} & \textsc{Noun} & \textsc{Verb} & \textsc{Adjective} & \textsc{Adposition} \\ \text{(6)} & \text{a. } \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} & \text{b. } \begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix} & \text{c. } \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} & \text{d. } \begin{bmatrix} -1 & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

Because Merge is taken to be an anti-symmetrical relation between a head (lexical item) and a phrase (or itself), the very first multiplicative operation in the system must be self-merge:

$$\text{(7)}\quad \text{a. } \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{b. } \begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{c. } \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{d. } \begin{bmatrix} -1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The results are identical: the familiar, very elegant, matrix *Z* Wolfgang Pauli used in his 1924 unpublished study on particle spin (https://en.wikipedia.org/wiki/Pauli_matrices). Orús *et al.* (2017) argue that, given the neutralization in (7) (all self-mergers lead to the same result), only one of the four outcomes may be semiotically relevant, which has to be chosen by axiom. They stipulate (7a) is the axiomatic anchor of human semantics; per Chomsky's assumptions, the self-merger of nouns. Once that is presumed, it can be shown that only the following combinations yield unique connections among all Chomsky categories (understood as categorial operators, with a "hat") and their putative associate phrases, their "complements", indeed in recursive terms (represented in the middle of the graph):

$$\text{(8)}\quad \underset{\mathrm{VP}(i)}{\begin{bmatrix} -1 & 0 \\ 0 & -i \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}} \xleftarrow{\widehat{\begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix}}_V} \underset{\mathbf{NP(-1)}}{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}} \overset{\widehat{\begin{bmatrix} -1 & 0 \\ 0 & -i \end{bmatrix}}_P}{\underset{\widehat{\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{-i} \end{bmatrix}}_N}{\rightleftarrows}} \underset{\mathrm{PP}(i)}{\begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}} \xrightarrow{\widehat{\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}}_A} \underset{\mathrm{AP}(1)}{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

In (8), by the Anchoring axiom, a self-merging noun $\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$ (**boldfaced**) plays a dual role: as an operator on itself, and with an argument of identical formal characteristics. Note how the Chomsky categorial operators take one of two "twin" matrices—in the sense that they (NP, VP, PP, AP) share the same eigenvalues and determinant (–1 for NP, *i* for VP, *–i* for PP, 1 for AP projections, as represented in parenthesis). The matrices aligned in the graph (proposed by physicist Michael Jarret) constitute an Abelian group. The Jarret graph makes the categorial generalization in (9). These correspond to more familiar examples as in (10); observe how (tail) recursive examples allowed in these basic terms include those in (11):

(9) *"Selection" conditions for operators in the Jarret Graph:*
   a. Nouns may either self-merge or take PPs to NPs.
   b. Verbs take NPs to VPs.
   c. Prepositions take NPs to PPs.
   d. Adjectives take PPs to Aps.

(10) *Exemplars covered by the Jarret Graph:*
- e. [$_{\text{NP}}$ *pictures* [$_{\text{PP}}$ *of* [$_{\text{NP}}$ *war*]]]
- f. [$_{\text{NP}}$ *hunt* [$_{\text{NP}}$ *rabbits*]]]
- g. [$_{\text{PP}}$ *of* [$_{\text{NP}}$ *war*]]]
- h. [$_{\text{NP}}$ *proud* [$_{\text{PP}}$ *of* [$_{\text{NP}}$ *science*]]]

(11) a. (... [*hear*]$_{\text{V}}$) [$_{\text{NP}}$ *relatives* [$_{\text{PP}}$ *of* [$_{\text{NP}}$ *friends* [$_{\text{PP}}$ *of* [$_{\text{NP}}$ *students* [$_{\text{PP}}$ *with* ... ]]]]]]
b. (... [*proud*]$_{\text{V}}$) [$_{\text{PP}}$ *of* [$_{\text{NP}}$ *stories* [$_{\text{PP}}$ *about* [$_{\text{NP}}$ *pictures* [$_{\text{PP}}$ *of* [$_{\text{NP}}$ *cities* [$_{\text{PP}}$ *with* ... ]]]]]]]

Selection conditions (9)/(10) do not cover all the possibilities sentences present: full (not just tail) recursion involving complex subjects, elaborate (di-)transitive structures, nuanced clausal embeddings... In order to extend to such possibilities, the system must go from the Jarret Graph, based on the Chomsky categories in (5a)/(6), into those (5b) presupposes, as well as symmetrical matrix conditions that extend the Chomsky categories in (5)—only diagonal matrices with mixed (real and complex) entries—by multiplying them by Pauli's matrix *X*, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, into matrices as follows (note the resulting anti-diagonal matrices in (b)):

(12) a. $\begin{bmatrix} \pm 1, \pm i & 0 \\ 0 & \pm 1, \pm i \end{bmatrix}$ b. $\begin{bmatrix} 0 & \pm 1, \pm i \\ \pm 1, \pm i & 0 \end{bmatrix}$

The ensuing set is a group for matrix multiplication, call it the Chomsky/Pauli group $G_{CP}$, which includes "grammatical categories". These do not start a derivation, they come on top of the "lexical structure" that the Jarret graph grounds. This can be expressed as in the super-Jarret graph (SJG) in Figure 1, presenting "extended projections" into functional structure, under an assumption I return to: that Hermitian matrices with only real eigenvalues (in light gray in the Figure) correspond to the basic semantic types (for entities, predicates, and truth values), while non-Hermitian matrices do to complex semantic types (all others). The dark gray category (C1, a mnemonic for Chomsky's first category in (6a)) correlates with the basic Anchoring Axiom (where computations start). Again, whatever semantics may result from these dependencies follows from, rather than determining, the algebraic fate of the graph.

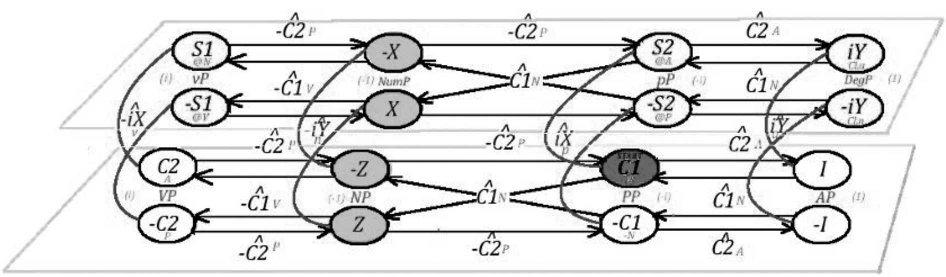


**Figure 1**

A (tentative) version of the SJG

**Table 1**

The "periodic table" of syntactic categories

| -1 ***Pauli Group*** | | | | *i* ***Pauli Group*** | | | |
|---|---|---|---|---|---|---|---|
| | | | | **Adjective** | **Adposition** | | |
| $Z = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{bmatrix}$ | $-Z = \begin{bmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$ | $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ | $-X = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ | $C2 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ | $-C2 = \begin{bmatrix} -1 & 0 \\ 0 & -i \end{bmatrix}$ | $S1 = \begin{bmatrix} 0 & 1 \\ -i & 0 \end{bmatrix}$ | $-S1 = \begin{bmatrix} 0 & -1 \\ i & 0 \end{bmatrix}$ |
| NP | | NumP? | | VP | | *v*P? | |
| | | | | **Pred** | ***C*** | | |
| $iI = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ | $-iI = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$ | $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ | $-Y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ | $iC1 = \begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$ | $-iC1 = \begin{bmatrix} -i & 0 \\ 0 & -1 \end{bmatrix}$ | $iS2 = \begin{bmatrix} 0 & i \\ -1 & 0 \end{bmatrix}$ | $-iS2 = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix}$ |
| NULL [–1]? | | PredP? | | NULL [–*i*]? | | DP? | |
| **1** | | | | ***–i*** | | | |
| | | a? | n? | Noun | Verb | | |
| $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | $-I = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ | $iY = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ | $-iY = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ | $C1 = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ | $-C1 = \begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix}$ | $S2 = \begin{bmatrix} 0 & 1 \\ i & 0 \end{bmatrix}$ | $-S2 = \begin{bmatrix} 0 & -1 \\ -i & 0 \end{bmatrix}$ |
| AP | | DegP? | | PP | | *p*P? | |
| | | ***p*?** | ***v*?** | **Det?** | ***T*?** | | |
| $iZ = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ | $-iZ = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$ | $iX = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ | $-iX = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$ | $iC2 = \begin{bmatrix} i & 0 \\ 0 & -1 \end{bmatrix}$ | $-iC2 = \begin{bmatrix} -i & 0 \\ 0 & 1 \end{bmatrix}$ | $iS1 = \begin{bmatrix} 0 & i \\ 1 & 0 \end{bmatrix}$ | $-iS1 = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix}$ |
| NULL [1]? | | TP? | | NULL [*i*]? | | CP? | |

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The SJG contains the Jarret graph as a sub-graph (in the lower tier), so that the information flow (what may be thought of as the *topology* of a syntactic derivation) can start in the emphasized node and go from any of the nodes in the "lexical" graph to this extension, where the observable/interpretable (Hermitian) categories are highlighted. Actually, the SJG—an empirical claim based on the formalism deployed, presuming the Jarret graph—contains only sixteen of the 32 categories in the $G_{CP}$, thus presupposing further nuances (related to the process of "movement" or Internal Merge). When all is said and done, the goal is to obtain a "periodic table" of categorial elements relevant to the syntax of natural language, as in Table 1, where the question marks suggest open issues.

In general, as with any "periodic table", one either needs to predict why a given gap exists in the paradigm or otherwise argue for a given underlying category "fitting the bill".

In the presumed terms, a lexicon is a network of relations built on what matrices as in (12) operate on: its own formal scaffolding "rotating" in four dimensions, in their algebraic combinations expressing the categorial distinctions Chomsky (1974) was after: nouns, verbs, etc., "up" to determiners, inflectional categories, and all that. The "states" (12) allows support lexical and grammatical categories involved in linguistic interactions. Evidently, assigning categorial features [+N, –V] or matrix $\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$ does not, in itself, distinguish all possible nouns there could be, so the implied algebra is meant to combine with whatever cognitive system we wish (vision, audition, the motor system...) for the further nuances one may want to add to the relevant vector space—the underlying syntactic scaffolding should still be what it is. This is just the algebraic foundation of the vector space where syntax lives, no more; but no less either: it allows for basic categories, their selection conditions, and interactions only some of which I touch upon in the next section.

## 4. Phrasal Interactions and Computational Conditions

The objects in the $G_{CP}$ are useful in one more regard, relating to the notion "computational space", where syntactic operations are supposed to happen. What sort of entity is this "space"? In contemporary linguistics, this is often discussed in terms of a kind of "workspace", like a blackboard or a calculator where certain elements are placed (a lexical array of some sort, some set of lexical tokens from the lexicon) and one starts "selecting" items-to-be-merged. In bottom-up systems that project phrases from lexical items, this way of thinking entails a further nuance, some sort of derivational "memory" that one must go into when invoking structures with phrasal complexity in more than one branch. This is easy to visualize if we combine (tail) recursive structures as in (11), in the following guise:

(13) $[_{NP}$ *relatives* $[_{PP}$ *of* $[_{NP}$ *friends* $[_{PP}$ *of* $[_{NP}$ *students* $[_{PP}$ *with delusions* ]]]]]]
*are* $[_{VP}$ *proud* $[_{PP}$ *of* $[_{NP}$ *stories* $[_{PP}$ *about* $[_{NP}$ *pictures* $[_{PP}$ *of* $[_{NP}$ *cities* $[_{PP}$ *with charm* ]]]]]]]]

While each of the separate lines in (13) can result from monotonic interactions in a single appeal to the Jarret graph, starting in the Anchoring Axiom (with self-merge of the noun category), in order to get the combined structure (full recursion) we must invoke the graph twice. In a standard Turing computational architecture, this is typically done by making use of the Turing memory tape for the elements that are not being processed, thus presuming a *current state* of the computation. For instance, one can proceed in the following steps:

(14) **Generate** [$_{NP}$ *relatives* [$_{PP}$ *of* [$_{NP}$ *friends* [$_{PP}$ *of* [$_{NP}$ *students* [$_{PP}$ *with delusions* ]]]]]]
(15) Place the contents in (14) **in derivational memory.**
(16) **Generate** *are* [$_{VP}$ *proud* [$_{PP}$ *of* [$_{NP}$ *stories* [$_{PP}$ *about* [$_{NP}$ *pictures* [$_{PP}$ *of* [$_{NP}$ *cities* [$_{PP}$ *with charm* ]]]]]]]]
(17) **Recall** [$_{NP}$ *relatives* [$_{PP}$ *of* [$_{NP}$ *friends* [$_{PP}$ *of* [$_{NP}$ *students* [$_{PP}$ *with delusions* ]]]]]] **and merge it to:**
(18) *are* [$_{VP}$ *proud* [$_{PP}$ *of* [$_{NP}$ *stories* [$_{PP}$ *about* [$_{NP}$ *pictures* [$_{PP}$ *of* [$_{NP}$ *cities* [$_{PP}$ *with charm* ]]]]]]]]

These dynamics are rather central in predicting, for example, so-called c-command conditions (holding within the monotonically generated structures and the top, or current state, of the stored phrasal chunks) or whether further syntactic processes are viable—generally not for portions that have been temporarily committed to working-memory. But the question now is whether this talk of "workspaces" or "memory" is literal or these are just ways of describing, in computational terms, something that is physiologically deeper. For example, if the notions are not metaphorical, does the system need a "clear" operation, as we need in calculators before we introduce a new multiplication?

Before delving deeper into this issue, it may be useful to compare two rather different notions of space that clearly arise in the arts: the one Michelangelo used in his studio and the one Beethoven invoked for composing. While at the end of the workday the artist's studio presented rubble and dust that his assistants had to clean up—and a new statue would require a further block—Beethoven's predicament was different, and not just when he was profoundly deaf and, thus, necessarily composing music "in his head". In a sense, all of music is "in one's head", since even playing with a melody in a given harmony, say on a string instrument, dissipates soon after contact between the musician and the instrument stops; there is certainly no "rubble" to be cleaned, once the energy of the sound waves disintegrates back to nature. In the latter instance, the "composition space" is quite abstract. Surely there is one, where one can tell whether a note is higher or lower, longer or shorter, to be more or less intensely "attacked", and so on; but there is little meaning to the idea of this space being different from the relations between sounds and rests themselves. The space *is* the set of relations, not some sort of tabula rasa where one literally places them. Are computational "workspaces" studio-like or like the space of interrelations in music?

The objects in the $G_{CP}$ suggest the latter answer, since it is easy to manipulate matrices as linear operators from a categorial to an interactive status, and vice-versa. To get the gist of how, consider a simple formal fact for all the objects in the group: self-multiplying them necessarily, sooner or later, returns the identity matrix $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

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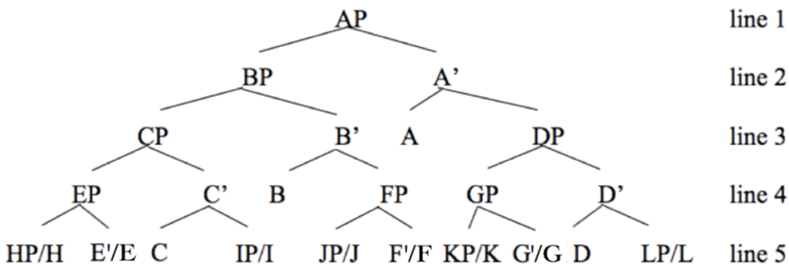
—this is what it means to be part of a multiplicative group. It may happen immediately, if one is self-multiplying *I* itself (but also any of the matrices in the group with just real entities in the diagonal), or eventually (in the odd powers if the entries in the diagonal are mixed or they are real but in the off diagonal; in further odd steps for mixed entries in the off diagonal). But there is no "growth" in these power sequences, any more than $1^n$ or $i^n$ grow regardless of the size of $n$: eventually such powers cycle back to 1, immediately in the real instance, after successive steps of –1, –*i*, 1, and *i* in the imaginary one. This "cyclicity" of powers for unit scalars, and comparable 2×2 matrices in the $G_{CP}$, using those scalars as entries in the diagonal or off diagonal, is useful in comparing that representational stability to the situation arising in less symmetrical matrices, where eigenvalues are not within the unit scalars, but are either smaller or larger, an asymmetry that results in a fractal structure (see Mandelbrot 1967).

A simple illustration of that point is through a method of mapping such asymmetric matrices to Lindemayer (L) systems that Medeiros (2012) discusses. While a Chomsky-grammar involves rewriting mechanisms applying one at a time, Lindenmayer (1968) eliminated this so-called Traffic Convention, so that in his formal systems each rewritable symbol must rewrite, at whatever derivational line the system is running. L-systems may be taken to describe the overall (maximal) topological space of a certain natural language expansion. For instance, Boeckx, Carnie & Medeiros (2005) show how the customary syntactic X'-theory yields maximally expanding structures as in (26) (the "slash" represents an unbounded expansion); simply counting the number of heads, intermediate projections, and maximal projections in each instance yields an obvious (generalized) Fibonacci pattern:

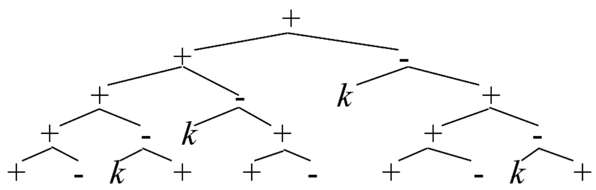
(19)




(20) a. Line 1: **1** XP
b. Line 2: **1** XPs + **1** X' = 2 categories
c. Line 3: **2** XPs + **1** X's + **1** $X^0$ = 4 categories
d. Line 4: **3** XPs + **2** X's + **1** $X^0$s = 6 categories
e. Line 5: **5** XPs + **3** X's + **2** $X^0$s = 10 categories
f. Line 6: **8** XPs + **5** X's + **3** $X^0$s = <u>16 categories</u> (predicted)
g. 13 etc. 8 etc. 5 etc. 26 etc. (predicted)

Those generalizations can be captured in matrix fashion by imagining how all relevant expansions, when seen combined, produces an L-tree following the Merge schema in (20) (where + is a maximal projection, - an intermediate one, and *k* a nonrewriting constant):

(21) a. Merge (+, –) $\rightarrow$ +
b. Merge (*k* +) $\rightarrow$ –




Then a matrix representation proceeds as follows (linear order irrelevant):

(22) a. Merge (+, –)
b. Merge (*k* +)
c. Unmerged *k*

$$\begin{array}{ccc} +\ \textsc{Tokens} & -\ \textsc{Tokens} & \boldsymbol{k}\ \textsc{Tokens} \\ \end{array}$$
$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

In Lindenmayer (maximally expanded) fashion, there is a matrix representation for the topology implied in (21) and the generalizations in (20) as seen in this sequence of powers:

(23) a. $$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^1 = \begin{array}{c} +\ \textsc{Tokens}\ -\ \textsc{Tokens}\ \boldsymbol{k}\ \textsc{Tokens} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$, corresponding to line 3 in (20c).

b. $$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$, corresponding to line 4 in (20d).

c. $$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^3 = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$, corresponding to line 5 in (20e).

d. $$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^4 = \begin{bmatrix} 5 & 3 & 2 \\ 3 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$, corresponding to line 6 in (20f).

f. $$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^5 = \begin{bmatrix} 8 & 5 & 3 \\ 5 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$, corresponding to a predicted line 7, etc.

The matrix representation neatly captures the fractal character of the L-system, as it phrasally grows, indicating how that general fractal growth proceeds. In contrast, an equal power sequence for the categories in $G_{CP}$ yields a cyclic return to the identity $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ only.

That the power sequence returns to the identity matrix or that it grows (as the case may be) is mathematically factual, depending on the magnitude of the highest

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eigenvalue in these square matrices: if it is 1, the system returns to the identity (describes a multiplicative group), if larger than 1, it grows as in (23), and if smaller than 1, it would shrink, in both instances in fractal fashion. The growth rate can be calculated via this eigenvalue; for the matrix in (22), the eigenvalues are $\frac{1}{2}(1+\sqrt{5})$, $\frac{1}{2}(1-\sqrt{5})$, and zero; the former two being the golden mean and its inverse (see Livio 2002). It can then be shown that the system (maximally) grows as a function of the golden mean, as ascertained by considering the aggregative series in (24). This corresponds to the (last) vertical series in (20) and is the third line in Wythoff Array (https://en.wikipedia.org/wiki/Wythoff_array), which organizes all Fibonacci series so as to cover the space of the natural numbers—without repetition.

(24) 2, 4, 6, 10, 16, 26, 42...

As I showed in 2018, (24) is also the first such aggregative series that can be decomposed into binary merge relations involving a single constant. Medeiros's matrix representation is a simple and useful way to analyze the system's formal properties.

In any case, psychologically and (possibly) neuro-physiologically, it is probably significant that the same formal representation (square matrices) allows us to speak of categories like those in the $G_{CP}$ and fractal phrasal topologies like those that (22) tidily represents (whose highest eigenvalue (growth signature) corresponds to the most elegant "continuous fraction" there is: 1+1/(1+1/(1+1/…)). Aside from the Galilean grace, one could relate multiplicative groups—computationally stable entities in that very repetition—to so-called earworms in the stuck song syndrome (https://en.wikipedia.org/wiki/Earworm). Again, the formal representation of matrix groups arising in power sequences disallows growth into a phrasal topology, thereby correlating with a kind of permanence for the algebraic structure. Surely that sort of stability is something one wants for categories, whose *raison d'être* is not to change every time they get deployed. Not the same issue for interactions, although there is still a sense, also, in which we don't want the basic interactive "rules of the system" to change with each use, at least not in terms of their underlying topology. The fractal expansions in (22) allow for that too: within its (expanded) confines, several sub-paths can be explored, as one does in the Pyrenees when following the underlying topology nature left for us, which is very far from a *tabula rasa*...

## 5. A Vector Space for Language and its Consequences

Importantly, the formal space where these categories and interactions exist is not separate from their existence—it is not a blackboard or a calculator where one adds numbers or clears them as relevant. Depending on the inner symmetries of the matrices, they are categories (highest eigenvalue of magnitude 1) *or* interactions (for magnitudes higher than 1, apparently the golden mean for the language faculty). This is not without substantive stipulations though, like the Anchoring Axiom that specifies for the topology of the Jarret graph to begin a computational life at the self-merger of nouns (not verbs or any of the others). In addition, it makes sense to ask about a further substantive stipulation depending on the nature of the eigenvalues, and whether they are measurable (real) or otherwise—in other words, corresponding to Hermitian matrices or not. In this, we are on the same boat as particle physicists, for whom these same operators, equipped with positive inner products (the central metric in a vector space), have real eigenvalues, which is key to representing physical quantities: measurements correspond to real quantities. Of course, and philosophically very interestingly, this separates what is "factual" (here, mathematically necessary for a system to work) from what is "observable" (or definitely identified for measurement). We may suppose this is how the abstract syntax interfaced other systems:

(25) *Interpretive Axiom*
Only Hermitian matrices within the $G_{CP}$ are legible at interfaces.

Again, (25) is not logically necessary, but it makes sense and has architectural consequences (primitive semantic types like "entity", "predicate", or "truth value correspond to the legible/real matrix projections), which one hopes has a neurophysiological correlate.

There are two such ideas I believe worth exploring. One is more or less implicit in the way (25) is stated, and it presumes a certain (porous) modularity to the linguistic system. On one hand, it should be relatively simple to connect a system with vectorial properties as we are assuming with others for which that reality is also presumed, starting (plausibly) with the visual and motor systems. I will not dwell on this here, beyond pointing out the obvious and signaling a formal consequence. That those other non-linguistic systems should be studied in a vectorial way is intuitive. Visually, not only do we need to process a luminous reality that can naturally be projected into 2D images in a retina; moreover, we can mentally rotate, skew, stretch, and distort such images, all of which are linear operations that vector spaces are designed to capture. Same with the motor system, particularly as it correlates with vision, as one grabs an object or catches a fly in midair, aside from (re-)calibrating our corporal stability to the ground, water as we swim, outer space in "zero gravity", and so forth. In my view, it is not surprising that vectorially-based XR games should be so successful, since they naturally reflect what our mind does best, particularly when placed in the quasi-embodied perspective of a helmet corresponding to our field of vision: then our, as it were, (0, 0, 0) coordinates in the physiological visual and motor systems are coopted by the device. Systems that speak the same language need no translation.

Then again, one must be careful about such interactions: we would not want contact with vision or the motor system to collapse formal distinctions in the faculty of language... There are ways to prevent that which I will only mention now: invoking (structure preserving) tensor products, which Orús *et al.* (2017) explore for phrasalization conditions summoning so-called specifiers. Naomi Feldman (in a cotaught seminar in Spring 2022) suggests that this sort of operation, between syntactic matrix categories and corresponding vectors in other mental systems, could yield successful connectivity without losing the presumed formal system, thereby keeping linguistic generalizations. Suppose in particular:

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(26) For $\pm A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$ a syntactic operator and

$\pm B = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix}$ an external vector,

$$\pm A \otimes \pm B = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \otimes \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} = \begin{bmatrix} a_{1,1}\begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{1,2}\begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \\ a_{2,1}\begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{2,2}\begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} a_{1,1}b_{1,1} & a_{1,1}b_{1,2} & a_{1,2}b_{1,1} & a_{1,2}b_{1,2} \\ a_{1,1}b_{2,1} & a_{1,1}b_{2,2} & a_{1,2}b_{2,1} & a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} & a_{2,1}b_{1,2} & a_{2,2}b_{1,1} & a_{2,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,2}b_{2,2} & a_{2,2}b_{2,1} & a_{2,2}b_{2,2} \end{bmatrix}$$

It should be easy to see, from the intermediate representation in (26), that the ensuing matrix (which after that step results in the less obvious distribution of values in the final matrix) contains the shape of the initial $\pm A$, even after the product. So: whatever this syntactic element was prior to this "connecting operation" *is still there*, in scaffolding. This is why I am arguing that the interaction of cognitive systems is modular: we have not lost the information that went into the connection, which by substantive condition (25), the Interpretive Axiom, we are limiting to only Hermitian matrices. At the same time, it is clear that, under these narrow conditions, the modularity is "porous", in that the systems are not prevented from "talking to one another", as implied in (26) and its results.

The second idea I want to signal pertains to what any of these uses of matrices may mean in neurophysiological terms. I will try to be clear when my suggestions here are speculative and when they are formally necessary, starting with the latter. Note, first, that diagonal $G_{CP}$ matrices as in (27) can be seen as reducing to column vectors as in (28):

(27) $\begin{bmatrix} \pm 1, \pm i & 0 \\ 0 & \pm 1, \pm i \end{bmatrix}$

(28) a. $\begin{vmatrix} \pm 1, \pm i \\ 0 \end{vmatrix}$, b. $\begin{vmatrix} 0 \\ \pm 1, \pm i \end{vmatrix}$

Matrix (27) operates on whatever vector space it is supposed to transform by carrying the "horizontal" or $x$ unit vector to (28a) and its "vertical" or $y$ unit vector to (28b). This is to scale that space according those conditions (which may be harder to visualize for the complex entries, but in that instance the spatial rotation can be seen as "going out of the paper"). The scaling in point can be in terms of whatever scalar we wish; we have been using $\pm 1$ and $\pm i$ entries, but any number $n$ or $m$ can perform this scaling, as we saw already in (23). This is to say, then, that the system can present decomposed operations as follows:

(29) a. $\begin{vmatrix} n(\pm 1, \pm i) \\ 0 \end{vmatrix}$, b. $\begin{vmatrix} 0 \\ m(\pm 1, \pm i) \end{vmatrix}$

And evidently either $n \neq m$ or $n = m$, the latter more symmetrically of course, in which case we can easily see the following equality (regardless of what $n = m$ is):

(30) $$\begin{bmatrix} n(\pm 1, \pm i) & 0 \\ 0 & n(\pm 1, \pm i) \end{bmatrix} = n \begin{bmatrix} \pm 1, \pm i & 0 \\ 0 & \pm 1, \pm i \end{bmatrix}$$

Once $n$ (=$m$) in the entries is seen as scaling the entire matrix, it is obvious that the matrix per se is in $G_{CP}$. This is useful in that conditions for elements in $G_{CP}$ are easy to come by.

Take for instance Hebbian "message coding", under the assumption that neuronal synaptic efficacy stems from neuron A exciting neuron B by persistently taking part in firing it, which is presumed to underlie *unsupervised learning* (under the (abstract) assumption that "neurons that fire together wire together"). Generally, that is correlated with positive scalars, but one can also imagine *inhibitory postsynaptic potential* (IPSP) interacting with *excitatory postsynaptic potentials* (EPSP) in real synapses, yielding negative entries. EPSP creates an excitable state at the post-synaptic membrane that has the potential to fire an action potential, while IPSP creates a less excitable state that inhibits the firing of an action potential by the post-synaptic membrane; physiologically, *IPSP is temporally superposed with EPSPs* to reduce the amplitude of the resultant postsynaptic potential, and of course +EPSPs and –IPSPs may cancel each other out when summed, a balance that seems key in integrating information at the synapse. One way to interpret what we are experiencing for objects in $G_{CP}$ is that, if respective scalars are symmetrically equalized leading to the situation in (30), then we get that kind of stability—for real entries. But how about the imaginary entries? Interestingly, Hebbian plasticity amplifies correlations in neural circuits, creating positive feedback loops, which lead to circuit instability unless something constrains them. It may be relevant that a chemical synapse's ability to undergo changes in strength (synaptic plasticity) is typically input-specific: the activity in a given neuron alters the efficacy of a synaptic connection between that neuron and its target, which is called "homosynaptic plasticity". There also is, however, "heterosynaptic plasticity", for which the activity of a particular neuron leads to input unspecific changes in the strength of synapses *from unactivated neurons.* It is thought that (aside to somehow contributing to the development of neural circuits and associative learning) heterosynaptic plasticity also relates to the homeostasis of synaptic input, curiously causing pathway unspecific synaptic changes *in an orthogonal direction* vis-a-vis Hebbian plasticity. This is so that whenever homosynaptic long-term potentiation is induced, unstimulated synapses should be depressed. Could this be literally "at right angles" in the intended sense, and modeled by complex entries in a system as we are pursuing?

This is where I am being speculative, but consider the matter abstractly. One notion that cuts across levels of representation in language is the opposition between

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punctual and distributed information. In phonology, this is obvious: a stop consonant is typically punctual, while a stressed vowel is just the opposite, and the former feature in the poetic phenomenon of alliteration (particularly at syllable *onsets*), while the latter do in the equally poetic phenomenon of rhyme (particularly at syllable, well, *rhymes*), then possibly holding across several verses. But phenomenologically we have a very similar opposition elsewhere in the linguistic system, for instance the difference between open-ended inner aspect in verbs (called "atelicity", as seen in *a professional runs for hours/#in several hours*) as compared to their terminal conditions (called "telicity", as seen in *a professionas reached the end in under four hours/#for hours*). Syntactically too, it is obviously not the same to interpret a name or demonstrative, as in *John saw that today*, than to interpret the verb in that expression. To put this in terms of a semantic representation as in Pietroski (2005):

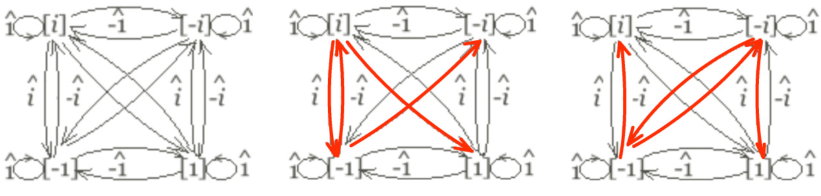
(31) a. *John saw that yesterday*
b. ∃*e*{Agent(*e*, *John*) ∃*e'*[Theme(*e*, *e'*) & Past-*see*(*e'*) & Theme(*e'*, *that*) & *today*(*e'*)]}

The interpretation of *John* (the agent), *that* (the theme), and the adverb *today* directly predicated of the event variable *e'*, are pretty punctual, which is to be instantiated precisely in the configurational context where they matter, as arguments or adverbial modifiers as the case may be. But the interpretation of the verb *saw*, associated to the sub-event *e'* that John causes, gets distributed over a "verbal frame" that has to be active throughout the processing of the entire proposition; it is as important to the temporal modification as it is to all the relevant arguments. Perhaps intuitively, neurophysiological events relating to the punctual stuff (the stop consonants, the *telos* for a lexico-semantic interpretation, the names and demonstrative "plugging" argument positions) should be, in some sense, less distributed than corresponding notions of an open-ended sort (vowels, atelic interpretation, verbal action). In the case of articulatory phonetics, we even know that punctual gestures are essentially "ballistic", while open-ended ones, engaging several muscles for a relatively long period of time, are "tetanized" instead. Something abstractly related is known in the perceptual literature too, called "active maintenance", normally associated to tracking denotations that may require sustained neural activity to hold them "in mind". My only point is that real eigenvalues in the $G_{CP}$ operators correlate more naturally with the punctual than the distributed stuff, which could be modeled through complex eigenvalues. If, in turn, the speculation about Hebbian postsynaptic potentials (+EPSPs and –IPSPs sums) vis-à-vis an orthogonal heterosynaptic plasticity is on track, the latter should be invoked in the neurophysiological correlate of tetanizations in the motor system or active maintenance perceptions of distributed information, modeling a distinction between the punctual and the distributed (akin to a particle/wave duality) that happens to be central to language.

Wolfgang Pauli demanded of scientific theories to at least be wrong. This one is not, at least in formal terms —it is what it is, with off-the-shelf linear algebra to churn computations. To test its empirical validity, though, we have to make substantive assumptions, like the Anchoring Axiom or these more speculative ideas about correlating Hermitian matrices with measurable/observable primitive semantic types (like names), as compared to the non-Hermitian matrices associated to complex semantic types. If the punctual/distributed distinction is seriously associated to neurophysiological phenomena like ballistic vs. tetanized/actively-maintained mental-events, a justification for the Interpretive Axiom could go from semantics to neurophysiology. The putative representation of a punctual notion in terms of a ballistic gesture and its interpretation, as compared to a distributed notion in terms of the tetanized/actively-maintained mental activity, should follow from the neurophysiological workings: the punctual nature of a simple synaptic event vs. the global reality of the complex brain dynamics sustaining tetanization or active maintenance. It could not have been the other way around, any more than rods could have been counterfactually used for color vision, while using cones for dark/light distinctions, also counterfactually—it is their very nature that leads them to the different, specific, function, in mammals and elsewhere in the animal kingdom. This would then be, once again, an instance in which the hypothesized interpretation in the linguistic system (whether semantic, phonetic, or anything else) tracks the natural architecture, instead of gearing it.

The Anchoring Axiom may also (partly) follow from these neurophysiological assumptions. Recall the neutralization of self-merger for the Chomsky matrices, as in (7). Why should the grammar decide that it is the self-merger *of nouns* (C1) that gears the Jarret graph? The very first step of self-merger presumes an operation from a non-Hermitian operator (as all the Chomsky categories are) whose results is Pauli's Hermitian *Z*. But why could we not have sanctioned this for the self-merger *of any of the other Chomsky categories*? Consider the possibilities the system presents, given possible matrix determinants (by examining the determinant products we are ipso facto contemplating the classes of multiplications that their associated matrices allow). Right away, we can determine that only the self-mergers landing on ±1 yield Hermitian matrix results to base the system on, limiting combinations to the lower part of the graph in (32a) (where 1 and –1 are represented as categories in brackets, not interactive operators with a hat)

(32) a. b. c.




We can also eliminate, on practical grounds, any of the combinations placing a category with determinant 1 at the core of the graph, because any phrasal merger-via-multiplication involving a matrix with determinant 1 (the identity or its negative counterpart) will endlessly yield the exact same result, which is not useful for a semiotic system. That said, the combinations in (32a) and (32b) still involve all four types of categories together with a recursive core landing on a Hermitian matrix, so the issue is why a system starting in Chomsky's four categories in (6) settles on (32c), the Jarret graph, as opposed to the alternative multiplicative possibility in (32b). Note that unlike (32c), which presumes the multiplication of –*i* by itself, (32b)

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forces the multiplication of *i* by itself (in both instances the result being –1). For this to yield a coherent, fully connected graph, we would need:

(33)
$$\underset{\mathrm{NP}(1)}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}} \xleftarrow{\widehat{\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}}_N} \underset{\mathrm{VP}(i)}{\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -i \end{bmatrix}} \underset{\widehat{\begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix}}_V}{\overset{\widehat{\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}}_A}{\rightleftarrows}} \underset{\mathbf{AP(-1)}}{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}} \xrightarrow{\widehat{\begin{bmatrix} -1 & 0 \\ 0 & -i \end{bmatrix}}_P} \underset{\mathrm{PP}(-i)}{\begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}}$$

This “evil-twin Jarret” graph corresponding to (32b) is perfectly sound algebraically, but it must stand in the self-merger of the Varrian/Chomskyan object that is hypothesized as an adjective. In a universe in which that were possible, selection restrictions would require adjectives to take VPs to project, while verbs take APs as their argument, then Ns terminally taking VPs and P’s terminally taking APs—obviously, nothing like real life. Of course, the Anchoring Axiom directly prevents (33), since it demands nouns to self-merge. But once we make the assumptions about some neurophysiology of categories as discussed above, it is plausible for the system to have to fall into a Hermitian matrix with determinant –1, which the “evil twin Jarret” graph associates to APs, not NPs. The proposed axiom reduction, then, is in terms of the formal system Varro hinted at and Chomsky, with the rest of the tradition, assumed as based on a verb/noun axis, not resting on adjectival/elsewhere categories. This is a deeper than the Anchoring Axiom, but it still leaves us with the homework of why it should be that the language faculty should be based on categories with a negative determinant, formally speaking (all of it, presuming the substantive choices that Chomsky, and Varro before, made—which could be challenged too, as everything can in science). In any event, the ensuing system, as the Jarret graph presents it, allows for punctual noun projections and correspondingly open-ended verbal projections, which suit well the denotation of (relatively) permanent entities vis-à-vis the more open-ended description of complex events. One could imagine, I am sure, an alternatively substantive semantics for relevant creatures based on the “evil-twin Jarret” graph. But seriously, would those be alien “hectapodes” as in the movie *Arrival*?

## 6. Conclusions

Categoricity in the present terms comes in two guises: observable and presumed, which I am hoping correlates with different forms of mental events, roughly punctual and, in that, directly measurable... vs. fundamentally distributed, which makes that open-ended. Both are equally factual and, indeed, essential to the algebraic architecture of the system. Both are, also, permanent in their inner symmetry, as seen by realizing that all the categories are part of the $G_{CP}$, a group for matrix multiplication. No matter how many times we multiply them (with themselves or with other matrices in the group), the result circles back to the identity—they cannot get interactive in a fractal (phrasal) sense, at least not by themselves. Then again, it is formally rather simple to pass from these permanent categories (be they particle-like or wave-like, measurable or otherwise) to phrasal conditions, so as to build a new thought: a mere imbalance in the system, of the sort arising when superposing (summing) these matrices, assuming normalization. In these interactions, a space of interrelations emerges in the categorial interactions, like Beethoven’s music.

I said I would return to the issue of substantive vs. formal distinctions introducing, in themselves, yet another categorization—which of course I have been presuming with gusto. The mystery here is not the substantive set of assumptions: that presumes simply the best available natural science. Our conception of heavenly bodies, their particles, life, or intelligence... is not like Aristotle's, or even quite Einstein's for that matter. Who knows what this century will bring; the issue is to try to stay curious, current, open-minded, rigorous, conciliatory, and so on. Whatever works, knowing that reality, as we uncover it, will always be way ahead our individual imaginations, though hopefully not too far ahead of our collective insight. Now the formal business is even weirder, crazily unreasonable (see Gödel 1962 [1931], to ponder to what an extent). Then again, by now, not admitting nature does speak the language of (some) mathematics amounts to not having gone to class, or to remain ideological for some reason, like insisting on not taking a proven vaccine. Whatever, but it just seems silly. Which is not to say anyone has the foggiest idea why the thing works *and* why some among us (lowly apes who cannot even keep from each other's throats while this beautiful planet crumbles) can tap into that gorgeous edifice, more or less the way Beethoven seems to have into the motifs of his late quartets.

Slightly less poetically, there is of course the issue of what makes us different from the rest of the apes. I will be the first to admit that we are, even if it may not seem so, often enough—at least in our potential and in those times any of us manages to convince the rest that some nugget of science, art, or simple decency matters enough to risk life and property or even loved ones to pursue it. I will not split hairs as to whether other mammals, birds, bees, octopi, or whoever else we like out there can actually share on some of those traits; say they do, to whatever extent seems demonstrable. It is still the case that we have put the Webb telescope out there, staring at the cosmos, and they have not, just as we could be responsible for killing us all, not they. So anyway: something happened, probably within the last half million years, possibly even less for what we naively call "anatomically modern humans". No one knows what, though it would be goofy to expect a visit from God, as most our ancestors believed. We blame evolution instead. Okay, but in what sense?

In that regard, one should bear in mind that natural philosophy is surely a recent event, no more than 2500 springs in (optimistically) half a million years of our existence, about a 0.5%. Formal developments presupposed here were not even imagined 250 years ago, the reason we call *i* imaginary (if anything I have said here is even remotely right, even just for particle physics, there is absolutely nothing *imaginary* to it!). 0.05% of our time as a species, in an optimistic scenario, is either terrifying or, actually, cause for some optimism. Either way, it should be seen as a reminder that our actual capacity for mathematics, as a species, is probably relatively less grounded than that of a Fields Medal winner, just as our musical capacity does not generally allow most of us to make a living as even street musicians. In short: it is not crazy to presume that whatever rudimentary algebraic base is actually general to the species is more or less what we use for the language faculty, the rest being culturally built on that when need emerges (probably not until well into the Neolithic).

All we can do is take those odds, still asking what brought us here, if nothing else to strengthen that path: the only one that gives us a fighting chance. So my crazy speculation: we can connect substantive and formal realities, assuming their very du-

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ality, pretty much the way we play as children. Our substantive reality, in neurophysiological networks, is what one gets in other species, give or take. We are not very remarkable in that regard, as our myths remind us: we cannot fly, breathe under water, use Earth's magnetic field to navigate, process infrared or ultrasound, and we have only two legs, not a thousand; we are weak, slow, noisy, and walk around naked, when most other mammals have gorgeous coats and control their environment in ways we would be too scared to think about. And yet... we can connect our pathetic substantive abilities in a way that gave us language; emphasis on *connect*. Apparently, some brain reorganization, by all accounts a rapid one (in evolutionary terms), allowed us to link unremarkable vocal learning abilities to unremarkable reasoning skills, which Minimalists think in terms of the language faculty providing an *optimal solution to an interface problem*. Importantly, this happened in such a way that not only could we bag concepts into atoms of sorts, but moreover we can somehow arbitrarily label them, via some noise or gesture that allows conspecifics to conjure them in their own mind. The rest is history. For some reason, no other known animal has been able to achieve this sort of result, although frankly it does seem as if some come close—and may still be able to do it in this planet, if we let it go on for a while.

That fairy tale is probably a bit more informed than others, even if only slightly so. My own twist, here, comes down to the idea that the way we "bag ideas" into categories is really not that different from the way in which those very categories get deployed as phrases. It is a matter of symmetry: as matrices, the categorial bags must be symmetrical (to be processed mentally, as earworms of sorts), while the phrasal fractal spaces that carry them as separate thoughts are fundamentally asymmetrical. The fact that both are extremely elegant, in Galilean terms, is probably nothing but the Chomsky ideal that all form in nature is Galilean, and language (like music) is pure form (see Chomsky 2005). In other words, this emphasizes the thought that the solution to the interface problem in the previous paragraph was, in a demonstrable sense, structurally optimal. That said, the dirty, but awesome, bit seems to be connective: the ability to relate those presumed levels of representation (it does not matter whether we call them phonology, semantics, or what, nor what they ultimately are, beyond empirical attestation) into some whole that we can collectively master, enough to build a common culture. In a nutshell: we do not seem to just accept the duality the universe gives us, between whatever substance it puts us in (like any other species, any other measurable entity really) and some form that, when connected, produces the magic. We transcend the duality by forcing a connection, *some* connection, good or bad. We insist on controlling how that substance marches on, just as Michelangelo did on seeking the vein in the marble, to imagine some David or other. Some do, anyway—and the rest of us try to follow.

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