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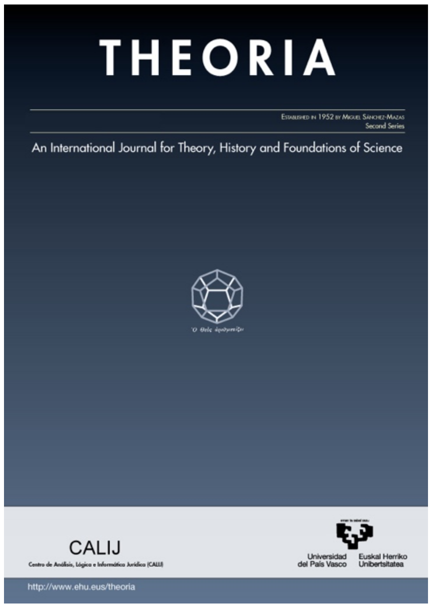


# Remarks on quantum mechanics and non-reflexive logic

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DOI: 10.1387/theoria.24771

Received: 13/04/2023

Final version: 12/02/2024

# Remarks on quantum mechanics and non-reflexive logic

*(Notas sobre la mecánica cuántica y la lógica no-reflexiva)*

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**Abstract:** In this paper, we discuss and outline a version of non-relativistic quantum mechanics based on a new non-reflexive logic, where the basic entities (elementary particles) lack identity conditions. Some relationships with quantum field theories are also sketched.

**Keywords:** quantum mechanics, quantum field theory, foundations, non-reflexive logic, lack of identity.

*Resumen: En este artículo presentamos y discutimos una versión no relativista de la mecánica cuántica, basada en una nueva lógica no-reflexiva, en la que las entidades básicas (las partículas elementales) carecen de condiciones de identidad. También se esbozan algunas relaciones con teorías cuánticas de campos.*

*Palabras clave: mecánica cuántica, teoría cuántica de campos, fundamentos, lógica no reflexiva, falta de identidad*

**Short summary:**

A new non-reflexive quantum mechanics is briefly presented, serving as a basis for future developments in the field of quantum foundations.

## 1. *Introduction*

Non-relativistic quantum mechanics (QM) was born in the late 1920s. It is the result of many physicists, especially Heisenberg and Schrödinger (cf. Jauch, 1968, Auyang, 1995).

The space and time of QM are the same as those of classical mechanics. So, QM is not relativistic in the sense that it doesn't obey the principles of special relativity. As a consequence, QM can neither treat the interaction of particles nor the relations between light and matter; radiation and other phenomena are also outside its domain. Notwithstanding, it explains numerous physical processes and its main ideas dominate the areas of atomic and molecular physics, as well as the foundations of chemistry since its birth.

In its common formulation, QM involves the notion of particles, for example, electrons. But this does not mean that particles are essential in QM: in the formulation of Heisenberg, as a matrix mechanics, particles are not involved.

It was necessary to construct a quantum field theory (QFT) in order to cope with problems connected with relativistic situations. QFT began with the work of Dirac in the late thirties on quantum electrodynamics, completed by, among other physicists, Feynman, Tomonaga, Schwinger, and Freeman Dyson. Later, QFT was extended to include the strong and the weak interactions.

Particles, in QFT, constitute only the epiphenomena of fields. QM can be considered, essentially, as a *limit* of QFT when the number of degrees of freedom of fields happen to be finite. In other words, QFT can be, approximately, reduced to MQ under certain special circumstances. QM in any of its formulations is of fundamental importance, from the physical point of view to QFT, more or less as classical mechanics is for QM (and even for QFT).

The concept of particle figures among the concepts of the standard Schrödinger's formulation of QM in which Hilbert spaces and Schrödinger equation are employed (see for instance the presentation in Sakurai, 1985). Nonetheless, electrons and other particles need not to be considered metaphysical entities, existing in themselves, as if they were substances. A less ambitious metaphysical stance remains possible: they would be, paraphrasing Russell, bundles of properties (on the connection between bundle theories and the ontology of quantum systems, see da Costa & Lombardi, 2014; da Costa, Lombardi, & Lastiri, 2013; Holik, Jorge, Krause, & Lombardi, 2022). Moreover, according to QFT, they would not exist as substances.

Anyhow, some physicists, like Schrödinger, argue that the notion of equality (or identity) cannot be applied to elementary particles, since this would be meaningless. Leaving aside space and time localisation, the properties of an elementary particle, for example of an electron, are well defined (mass, charge, ...) and characterise the *family* of electrons, but not a particular electron; this is true of any class of similar elementary particles (I shall not expose here the conceptions of Schrödinger; the interested reader may consult, for instance, Bitbol, 1996).



Schrödinger's ideas brought into my mind, almost forty-five years ago, the following questions: (1) it is possible to develop a logic in which identity is restricted, because it is meaningless for certain objects? (2) is there a formulation of QM in which the elementary particles are not subjected to equality? (cf. da Costa, 1980).

The next two sections of the present paper will be concerned with the two above questions.

## *2. Non-reflexive logic*

The first problem is not easy. Anyhow, let us call non-reflexive a logic $\mathscr{L}$ in which the relation of equality (or identity) does not make sense for some or for all objects of the domain of $\mathscr{L}$. If this is so, then there exist objects that do not obey the Principle of Identity or Reflexive Law of Identity,

$$x = x \tag{1}$$

or

$$\forall x(x = x). \tag{2}$$

It seems at first sight almost impossible to give a semantics, formal or informal (intuitive), for logics such as $\mathscr{L}$.

Classical logic, say, had an informal semantics even before today's usual set-theoretical semantics, introduced especially by Tarski. Notions like *proposition*, *denotation*, *truth*, *term*, etc., were intuitively known; the set-theoretical semantics constitute a 'mathematisation' of the intuitive stance. However, the set-theoretical approach, *per se* cannot give *real* meaning to the classical logical operators, since it is based on a set theory that, to be constructed, needs already classical first-order logic.

The situation is not the same regarding $\mathscr{L}$. An informal semantics can only be described in everyday language plus possibly some extra terms. But without the help of equality, there is no way to build such (informal) semantics. In fact, for such a task it is needed to appeal to words such as *other*, *different*, *another*, *same* and *unique*, as well as to expressions similar to '$x$ denotes one and only one object' and '$x$ is the same object as $y$', or expressing discrimination of one thing from another. Thus, we get implicitly or explicitly involved with equality.

On the other hand, a formal semantics analogous to the Tarskian semantics for classical logic offers serious obstacles to be accomplished. Since classical logic has a starting intuitive semantics, while non-reflexive logics doesn't, the nature of the two kinds of logic are intrinsically different.[1]

Nonetheless, the problem possesses a reasonable solution: Curry, in essence, proposed that the syntactical rules for handling the logical symbols could be used to give a syntactical meaning to them (Curry, 1957, 1963). Gentzen's formulation of the basic laws and rules of logic would suffice for this objective (Szabo, 1969, Kleene, 1952, Chap. XV).

If the rules governing the symbols of $\mathscr{L}$ are analogous to those of classical logic, then Curry's view remains valid, including the equality symbol when it can be employed.

In effect, if we focus our attention on Gentzen's formulation of extant logics, then it is clear that the rules of introduction and discharge of logical symbols grant certain meaning to them. Even the axioms or schemes of axioms can be considered as contributing to the syntactic meaning of the primitive symbols. In synthesis, the formal structure of the logic $\mathscr{L}$ (and of the corresponding deductive systems based on $\mathscr{L}$) implicitly determine the meaning of the operations and postulates (including the primitive rules) of $\mathscr{L}$.

Below we formulate a strong non-reflexive logic that is easily translatable into Gentzen form. Other such logics may be found in da Costa and Krause (1994, 1997).

### *3. A new non-reflexive logic*

We say that equality has no meaning in a given context $\mathcal{C}$ if and only if the Principle of Identity is not valid for the objects (or part of them) belonging to the domain of $\mathcal{C}$.

The non-reflexive logic $\mathscr{L}$ we have in mind is formulated as follows: (1) we introduce the syntactical notion of *type*; (2) the concepts of primitive terms, term and formula are defined; (3) the main postulates of $\mathscr{L}$ are presented (others could also be added).

[1] An anonymous referee pointed out that one may present a semantics for non-reflexive logical systems in terms of extant quasi-set theory (see French & Krause, 2006; Krause, 2023; Krause & Arenhart, 2018). While this is true, it is hard to present an *intuitive* semantics for non-reflexive logics, and also quasi-set theory itself.

The set $\mathsf{T}$ of *types* constitutes the least set satisfying the following conditions:

1. The symbol $i$ belongs to $\mathsf{T}$
2. If $a_0, \ldots, a_{n-1} \in \mathsf{T}$, then $\langle a_0, \ldots, a_{n-1} \rangle \in \mathsf{T}$, with $1 \leq n < \omega$ and $\langle a_0, \ldots, a_{n-1} \rangle$ being the finite sequence of $n$ items, composed by $a_0, \ldots, a_{n-1}$.

The language $L$ of $\mathscr{L}$ has the following primitive symbols:

1. Connectives: $\rightarrow$ (implication), $\wedge$ (conjunction), $\vee$ (disjunction), and $\neg$ (negation); $\leftrightarrow$ (equivalence) is defined as usual.
2. Quantifiers: $\exists$ (there exists) and $\forall$ (for all).
3. For each type, a denumerably infinite set of variables of this type.
4. For each type, a family of constants of this type; some families may be empty.
5. The equality (or identity) symbol, $=$. It will be of type $\langle i, i \rangle$.
6. Parentheses.

The notions of term (of type $a$, $a \in \mathsf{T}$), formula, sentence (formula without free variables), free term for a variable in a formula, etc. are easily defined in the usual way. We only note that $\mathscr{L}$ is a higher-order logic and that the atomic formulas of $L$, the underlying language of $\mathscr{L}$, are subjected to the standard restrictions of type theory.

We now list the postulates of $\mathscr{L}$ (axiom schemes and primitive rules of inference). In the writing of terms and formulas, we always suppose that the restrictions of types are observed. Our notations are those of Kleene (1952), adapted to higher-order logic, including obvious changes.



**Postulates of $\mathscr{L}$.**

1. The complete system of postulates for the classical propositional calculus of Kleene, 1952.
2. The following schemes and rules of the same Kleene, 1952, conveniently adapted to higher-order logic (with restrictions analogous to those of Kleene, 1952.

$$\forall x A(x) \rightarrow A(t), \qquad A(t) \rightarrow \exists x A(x) \tag{3}$$

$$\frac{A \rightarrow B(x)}{A \rightarrow \forall x B(x)}, \qquad \frac{A(x) \rightarrow B}{\exists x A(x) \rightarrow B} \tag{4}$$

**Definition 1.** *If u is a term and v is a variable, both of type i, then:*[2]

$$\mathscr{I}(u) := \exists v(u = v). \tag{5}$$

[2] Following the reading of Schrödinger's works, we separate quantum entities from classical entities, with the former instantiating restrictions on the relation of identity. As a result, this predicate singles out terms referring to entities for which identity 'makes sense'. Notice that this is different from typical strategies for non-reflexive logics (e.g. in da Costa and Krause, 1994, 1997 and French and Krause, 2006, Chap. 7), where non-reflexivity is encapsulated by the fact that identity does not compose a well-formed formula for some terms. The plan here is that although identity is not restricted in the syntax, it does not have any kind of inferential effect, unless the entity instantiates $\mathscr{I}(u)$; this idea is reflected in the postulates for identity, relativised to $\mathscr{I}(u)$.

By considering this definition, we introduce the postulates of equality, with usual restrictions in (4):

1. $\mathscr{I}(u) \to u = u$
2. $\mathscr{I}(u) \wedge \mathscr{I}(v) \to (u = v \to v = u)$
3. $\mathscr{I}(u) \wedge \mathscr{I}(v) \wedge \mathscr{I}(w) \to (u = v \wedge v = w \to u = w)$
4. $\mathscr{I}(u) \wedge \mathscr{I}(v) \to (A(u) \leftrightarrow A(v))$

The *postulate of abstraction* is put this way, with the usual restrictions:

$$\exists P \forall x_1 \ldots \forall x_n (P(x_1, \ldots, x_n) \leftrightarrow A(x_1, \ldots, x_n)). \tag{6}$$

**Definition 2** (Equality relative to a family of properties)**.** *If $B$ is a term of type $\langle\langle a\rangle\rangle$, $P$ is a variable of type $\langle a\rangle$, and $X$ and $Y$ are terms of type* a*, then:*

$$X =_B Y := \forall P (B(P) \to (P(X) \leftrightarrow P(Y)). \tag{7}$$

**Definition 3** (Of equality relative to a type)**.** *If $X$ and $Y$ are terms of type* a *and $P$ is a variable of type $\langle a\rangle$, then*

$$X =_{\langle a\rangle} Y := \forall P (P(X) \leftrightarrow P(Y)). \tag{8}$$



The relation $=$ may be called *absolute equality* ) or identity), while $=_B$ and $=_{\langle a\rangle}$ are relative equalities (or identities).

Some extra, well-known postulates could be added to the above list; for instance, certain formulations of the Axiom of Choice and the Axiom of Infinity. Moreover, depending on the objectives of $\mathscr{L}$, (such as, say, to be the basic logic of an extant non-relativistic quantum mechanics).

One of the possibilities open to $\mathscr{L}$ is to adjoin to it a symbol of membership of type $\langle i, i\rangle$, and to develop a higher-order set theory. This way, among other things, we can construct inside $\mathscr{L}$ models of $\mathscr{L}$, to build a theory of models in $\mathscr{L}$ and to show that $\mathscr{L}$, if consistent, is incomplete.

A profound modification of $\mathscr{L}$ is the following: we impose that the relation of (absolute) identity makes sense only for some individuals. In fact, this would perhaps be Schrödinger's intuition, at least in his 1952 book; this point of view was exploited in da Costa and Krause (1994, 1997).[3] However, in conformity with some present-day physicists, the same quantum objects can sometimes act as macroscopic objects (for which equality can be applied) and sometimes as microscopic objects (to which equality does not apply); this is the case, for example, of Zurek (2009). Anyhow, questions on the relations between logic and physics will be left to the next section. Our aim in this section was only to show that there are, in principle, strong non-reflexive logics (i.e. a higher-order logic).

[3] See also French and Krause (2006) and Krause (2023).

## *4. Logic and quantum mechanics*

The inner nature of QM, in its usual formulation, is that it is a theory of particles of special sort. QFT encompasses QM, although it constitutes a theory of fields. We can pass, so to say, from QFT to QM when the degrees of freedom of a quantum system are finite in number. QFT may be, in various situations, reduced to QM. Auyang asserts that

> Normal modes, field quanta, and particles are good concepts for describing continuous systems only when the coupling between them is negligible. The condition is not always satisfied. For instance, the modes of a violin string cannot be regarded as independent of each other when the vibration is violent enough to become anharmonic. Similarly, when quantum fields interact, quanta can be excited and deexcited easily so that the static picture of free fields depicted above no longer applies. That is why field theorists say particles are epiphenomena and the concept of particles is not central to the description of fields. (Auyang, 1995, p. 53)

Nonetheless, QM is a theory of particles, at least in its usual formulations. Thus, there exists some tension between QFT and QM. However, QM is of fundamental significance for QFT: without QM there would be no QFT. The case is analogous to that between classical physics (and, in particular, classical mechanics) and QM; the former can be methodologically separated from the latter.

After Born's interpretation of the wave function as furnishing us probabilities of states of particles or collections of particles, there seems to be no way to leave aside particles in QM.

Clearly, the particles of QM are not particles of classical mechanics (cf. Falkenburg, 2007, Chap. 5). Anyhow, they are essential to QM. We employ, then, incompatible theories in the domain of physics, what a paraconsistent logic can explain (see da Costa, 2007). Similar situations occur commonly in physics. As Wick says,



> This [20th] century began with a dilemma and a paradox. Two great theoretical paradigms—Newton's mechanics and Maxwell's electromagnetism—each supporting a splendid structure but contradictory at the join, formed the dilemma. The twin pictures of wave and particle cast the paradox. Now as we approach the century's end, despite all the successes, we face, strangely, an almost identical situation. Two successful theories, general relativity and quantum mechanics, are triumphant in their own realms, yet remain strangely silent across their mutual boundary. And our old friend, the paradox of the continuous and the discrete, remains. (Wick, 1995, p. 199)

In QM each class of particles has its definite intension, i.e. the collection of the central properties of its elements. The electron, say, is characterised by its mass, electric charge and spin; space and time properties are not, in a sense, indispensable to treating its dynamic behaviour. So, one could try to explore this state of affairs by means of a non-reflexive logic: the elementary particles, atoms and molecules would stay out of the reach of equality; it would be, for example, meaningless to talk, for example, about equal or different elementary particles. We could, then, try to see what portion of QM could be treated without equality.

Such research is possible and this is how we interpret some works, such as those of Krause and others (see, for example, de Barros, Holik, & Krause, forthcoming; Domenech, Holik,

& Krause, 2008; French & Krause, 2006; Krause, 2023; Krause & Arenhart, 2018).

Anyway, the underlying logic of QM is classical logic, including equality. This relation, then, may be envisaged as an ideal relation in QM: even if we cannot, or should not, operationally distinguish or identify two particles, nonetheless they remain, in principle, identical or different. For instance, in the statistics of Bose–Einstein or of Fermi–Dirac, although the probabilistic measures *identify* particles, this fact does not require that equality be meaningless for them.

QM, in its extant formulations, is one of the most successful branches of knowledge of all time, explaining a great number of phenomena. So, any tentative modifications of it, inside its field of applications, must have a strong motivation.

In what follows, it is sketched a new formulation of QM based on non-reflexive ideas.

## 5. *Non-reflexive quantum mechanics*

To begin with, we must explicate what we really understand by QM and, to attain our objective, the sole procedure is to appeal to the axiomatic method, here conceived in a wide sense, although we proceed intuitively.

Even employing the axiomatic method, we face problems. For example: 1) Should the experimental counterpart of QM, say the double slit experiment, be included in our axiomatisation? 2) Should lasers and masers also be included? (These problems and many others of similar nature concern the empirical part of QM).

If the experimental setting of QM is to be considered as part of its axiomatic nucleus, then our task would be herculean (the usual axiomatisations of QM don't take its empirical level into consideration).[4]

So, let us fix our attention to one of at least partial axiomatic systematisations of QM, for example, that of Faris, here denoted $\mathcal{F}$, as it appears in Wick (1995), and of which we omit the details. The developments below remain valid when other axiomatisations are used instead. Thus, from now on, we perfectly know what we are talking about: QM strictly means Faris' axiomatisation.

QM has, as one of its bases, classical mathematics.[5] Therefore, $\mathcal{F}$ is founded on ZFC (with *Urelemente*), plus some extra terms belonging to QM and having a collection of specific postulates formulated by Faris.

The following definition is central: if $A$ is a sentence (of the language) of $\mathcal{F}$ that does not explicitly contain the symbol of equality between two constants or variable terms designating quantum particles, then $A^*$ will denote $A$; if $A$ contains occurrences of the equality symbol relating terms denoting quantum particles of class $P$ (see the previous section), then we replace in $A$ each such occurrence by $=_P$; under this hypothesis, $A^*$ is obtained by making all the replacements in $A$. The set of all sentences $A^*$ such that $A$ is valid in $\mathcal{F}$ constitutes the theory $\mathcal{F}^*$.

[4] What is meant by 'experimental setting', as the paragraph makes clear, is the application of the theory to particular scenarios and experiments. These applications may be present in the *heuristics* for the axiomatization, but the axioms themselves are meant to structure only the mathematical apparatus. For a discussion on how the formal apparatus relates to the empirical world, see Krause (2024).

[5] As an anonymous referee pointed out, at least *standard* quantum mechanics has. Of course, QM can also be reformulated based on quasi-set theory (see Krause & Arenhart, 2016); in particular, using a Fock-space approach (see Krause & Arenhart, 2018).

$\mathcal{F}^*$ is closed by classical logic and it does not contain any sentences involving equality between quantum particles: $\mathcal{F}^*$ may be seen as a kind of non-relativistic quantum mechanics in which there is no equality relating quantum particles; in other words, it is a non-reflexive theory.

Furthermore, the consistency of $\mathcal{F}$ implies that of $\mathcal{F}^*$.

We cannot talk about equal or different particles in the empirical part of $\mathcal{F}^*$. For instance, in the double slit experiment, when a particle is emitted from a source, passes through one or the other hole and is detected, it would be meaningless, strictly speaking, to assert that the particle emitted and afterwards detected are the same or different.

If we add to $\mathcal{F}$ (and consequently to $\mathcal{F}^*$) an axiom asserting that all objects are micro-objects or macro-objects, then $\mathcal{F}^*$, so modified seems to be essentially the non-reflexive theory of Krause (see de Barros et al., forthcoming; Krause, 2023).

$\mathcal{F}^*$ is a strong theory. It follows that identity can be removed from a good portion of QM. This state of affairs contributes to reinforcing the thesis that some forms of realism (sometimes with locality) are not compatible with QM, a topic related to the problem of hidden variables.

It must be clear that we are not proposing to abandon extant QM, but only trying to better analyse, from the methodological point of view, its logical structure. In effect, at the moment, it would be silly to intend to modify QM. Nonetheless, this does not mean that QFT is irrelevant.

As Wick writes,



> Quantum mechanics was one of the most successful creations of the human intellect. From the color of neon lights to the hardness of diamonds to the magnetism of electrons, it correctly described a host of physical phenomena. When it worked, it worked. But quantum mechanics was not the end of physics. (Wick, 1995, p. 200)

However, we should not forget that QM has its own limitations, which are absolutely patent to everyone knowing it. As Zee declares,

> Write down the Schrödinger equation for an electron scattering off a proton. The equation describes the wave function of one electron, and no matter how you shake and bake the mathematics of the partial differential equation, the electron you follow will remain one electron. But special relativity tells us that energy can be converted to matter: If the electron is energetic enough, an electron and a positron ("the antielectron") can be produced. The Schrödinger equation is simply incapable of describing such a phenomenon. Nonrelativistic quantum mechanics must break down. (Zee, 2010, p. 3)

We lay stress on the fact that QFT copes with problems involving special relativity parameters. In other words, as Zee says,

> [...] [QFT] was born of the necessity of dealing with the marriage of special relativity and quantum mechanics, just as the new science of string theory is being born of the necessity of dealing with the marriage of general relativity and quantum mechanics. (Zee, 2010, p. 6)

One of the interesting characteristics of $\mathcal{F}^*$ (and QM) is that it doesn't distinguish, in principle, between micro-objects and macro-objects. Thus, it is open to the possibility that all objects of QM and $\mathcal{F}^*$ belong to the same category, that of quantum objects (on this subject, see Ball, 2008, which makes particular reference to the quantum Darwinism of Zurek, 2009).

## *6. Final remarks*

We intend to develop the main ideas of the preceding outline in future works.


## *Acknowledgments*

We thank R. Klipert and D. Krause for their criticisms and advice. We acknowledge the partial support from the Brazilian National Research Council (CNPq).


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