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A truthmaker-free approach to arithmetical truth

(Una aproximación sin verifactores a la verdad aritmética)

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Abstract: Truthmaking is usually defined as a relationship of ontological dependence that holds between propositions (the truthbearers) and worldly objects (the truthmakers). But is this accurate? If we have reasons to answer this question in the negative, then we can either allow other kinds of objects as truthmakers or we can go further and state that truthmaking does not require a supposition that any particular kinds of objects are truthmakers, leading to a sort of ontological deflationism about truthmaking. These answers have already been raised in the literature as genuine possibilities. Still, none of them has been substantiated by anything more elementary and fundamental than truthmaking, which might render such responses ad hoc. In this paper, it will be shown that by relying on grounding, we can support both approaches. To demonstrate this, we will use arithmetical truth as the backbone of the paper, showing that we have independent reasons both to allow abstract objects to be truthmakers for arithmetical statements and to dispense altogether with the reliance on truthmakers to explain their truth.

Keywords: Truthmaking, Grounding, World-to-Truth Thesis, Structural and Logical Principles, Arithmetical Statements, Ontological Deflationism.

Resumen: La verifacción (truthmaking) suele definirse como una relación de dependencia ontológica que se da entre las proposiciones (los portadores de verdad) y los objetos del mundo (los verifactores). Pero ¿es esto correcto? Si tenemos razones para responder negativamente a esta pregunta, entonces podemos o bien permitir que otros tipos de objetos sean verifactores, o bien ir más allá y afirmar que la relación de verifacción no requiere presuponer ninguna clase de objetos como verifactores, lo que nos lleva a un tipo de deflacionismo ontológico respecto a la verifacción. Estas respuestas ya han sido planteadas en la literatura, pero

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ninguna de ellas ha sido sustentada en algo más elemental y fundamental que la propia noción de verifacción, lo que sugiere que tales respuestas sean ad hoc. En este artículo se mostrará que, al basarnos en la noción de fundamentación (grounding), podemos apoyar ambos enfoques. Para ello, se usará la verdad aritmética como eje central del trabajo, mostrando que tenemos razones independientes tanto para permitir que los objetos abstractos sean verifactores de enunciados aritméticos, como para prescindir por completo de verifactores para explicar su verdad.

Palabras clave: Verifacción, Fundamentación, Tesis del Mundo a la Verdad, Principios Estructurales y Lógicos, Enunciados Aritméticos, Deflacionismo Ontológico.

Short summary: Truthmaking is typically viewed as an ontological dependence relation between truths and worldly entities; however, this view might be flawed. Using arithmetical truth, the paper argues that grounding supports two non-ad hoc alternatives: allowing abstract objects as truthmakers for arithmetical truths or rejecting the need for specific truthmakers altogether, thus advancing a form of ontological deflationism about truthmaking.

1. *Introduction*

An important question arises in the semantics of arithmetic: *What kind of entity*—if there is one—*explains the truth of an arithmetical statement?* This question implies the analysis of two relevant issues:

(1) The first issue concerns truthmaking. Suppose we assume that truth is not primitive, but that what is true depends upon what is in the world. In that case, we should embrace the World-to-Truth thesis (**WT**), according to which there must be entities in the spacetime world (truthmakers) that account for the truth of arithmetical statements (Bigelow, 1988; Armstrong, 1997, 2004; Rodriguez-Pereyra, 2005; Asay et al., 2019).

(2) The second issue deals with grounding. It is plausible to claim that an explanatory statement is concerned with the grounding of something (the *explanandum*) in something else (the *explanans*) (Merricks, 2007; Rosen, 2010; Schnieder, 2011; Fine, 2012a; Littland, 2015; Dasgupta, 2017).

Because of this last point, some have argued that, since explanation is a type of grounding, and (**WT**) is an explanatory statement, truthmaking should be interpreted as a kind of grounding (Correia, 2005; Rodriguez-Pereyra, 2005; Schnieder, 2006; Caputo, 2007; Trogdon, 2020). Note, however, that it does not follow from this that truthmaking can be *reduced* to a kind of grounding, where by "*reduced*" we mean that it could be a type of grounding like any other, with no *prima facie* restrictions regarding its *relata*. If we interpret (**WT**) literally—as Armstrong (1997, 2003, 2004, 2007) and Bigelow (1988), among others, in fact do—the most we could claim is that truthmaking is a *restricted* type of grounding in which only propositions (or, more broadly, linguistic entities) and worldly objects are taken to be *relata* (Audi, 2012, 2020; Heil, 2016; see also Mulligan et al., 1984; Fox, 1987; Bigelow, 1988; Armstrong, 2004; Fine, 2017).

However, there is a more substantive sense in which the relationship between truthmaking and grounding can be grasped. To understand this perspective, one must first realize that, at first sight, truthmaking and grounding are two different procedures. As previously pointed out, truthmaking is traditionally defined as a kind of ontological dependence between two ontological categories: propositions and worldly objects (Rodriguez-Pereyra, 2005). Grounding is also a type of ontological dependence, generally considered an intracategorial relation (Rosen, 2010; Audi, 2012; Fine, 2012a; Schaffer, 2012; Dasgupta, 2014; Krämer et al., 2015; Dorsey, 2016; Correia, 2017, 2020; Thompson, 2018; Maurin, 2019). If so, however, this constraint will not result from any pre-existing restriction on the grounding relation (Fine, 2001; Correia, 2010). Accordingly, certain advocates of grounding have suggested that no aspect of this relationship prevents it from being viewed as an intercategorial relationship (Schaffer, 2010a; deRosset, 2013). Consequently, they contend that the grounding relation can hold between entities belonging to any ontological category, and notably, not solely between propositions and worldly entities.

Hence, if we could somehow argue that truthmaking is not just a restricted form of grounding but can be reduced to it (and thus truthmaking would be a type of grounding, *tout court*), we would be asserting something new about truthmaking: that, contrary to what most truthmaker theorists claim, the kinds of objects that can play the part of truthmakers can be other than worldly objects because the restriction could be removed. Consequently, this new perspective would allow us to define "truthmaking" as a kind of grounding relation that holds between a linguistic object and an object, but not necessarily a worldly object. We will return to these points in §3.

In §2, a reduction of truthmaking to grounding will be undertaken, arguing that, in a specific sense, truthmaking can be understood as an unrestricted type of grounding concerning the entities it takes as truthmakers. This reduction enables us to derive two distinct answers to the initial question. The first answer is that we can acknowledge mathematical objects as truthmakers because grounding can allow this type of entity to ground arithmetical truths (§3). The second is that, if grounding can be understood as a relation of ontological dependence that does not require a commitment to any particular entities as grounds (Melia, 2005; Schnieder, 2011; Dasgupta, 2017; Kovacs, 2018), then a similar outcome could be extended to truthmaking; thus, we would not need truthmakers to make arithmetical statements true (§4).

These two responses to the initial question represent the central philosophical aims of this paper because they offer two distinct explanatory accounts of arithmetical truth. Of the two, the second is more conceptually radical. However, this viewpoint does not imply that it is not useful or even necessary to engage with numbers or other mathematical entities. This approach simply maintains that such entities are not required to account for the truth of arithmetical statements (§5). Therefore, the idea is to adopt a deflated account of truthmaking—in which there can be truthmaking without the need to posit truthmakers-—as a working hypothesis for explaining arithmetical truth and investigate whether more substantial metaphysical commitments are needed.

It will be understood by "ontological deflationism" the view that alleged ontological issues—such as truthmaking—are not substantive from an ontological point of view but can be reduced to semantic, empirical, logical, or other problems that do not involve genuine ontological inquiry (Thomasson, 2014). In contrast to the relationship between truth deflationism and truthmaking,[1] the debate over whether truthmaking can be understood in terms of ontological deflationism has received little attention.[2] This paper aims to address this gap.

2. *From Grounding to Truthmaking*

One way to reduce truthmaking to grounding is to first define structural and logical principles that provide an exhaustive characterization of both notions, and then derive truthmaking principles from these grounding principles. This is the strategy suggested by Correia (2014).

2.1 *Some Grounding and Truthmaking Principles*

Let us begin by establishing some notation, which is the same as that used by Correia (2014). First, the symbol "$\triangleright$" is employed to express the grounding relation that holds between a non-empty list of sentences Δ and some well-formed expression φ. Thus, "$\Delta \triangleright \varphi$" can be read as "$\Delta$ *grounds* φ." Additionally, we take X to be a list of one or more object names $a_1, a_2, ..., a_n$. "$\models$" stands for the "making true" relation, so "$X \models \varphi$" can be read as "X makes it true that φ." $E!X$ is a list of sentences of the form a_1 exists, a_2 exists,..., a_n exists. Finally, we will incorporate a disquotation operator "$[\![\,]\!]$" into the language that operates on well-formed expressions, along with a truth predicate T that carries a semantic interpretation based on Tarski's framework:

$$T([\![\varphi]\!]) \textit{ iff } \varphi \qquad (\textit{Tarski}^{\leftrightarrow})$$

Henceforth, we will write $T\varphi$ instead of $T([\![\varphi]\!])$. Using this notation, we can provide a widely accepted[3] formulation of the account of truthmaking in terms of grounding:

$$X \models \varphi \textit{ iff}_{df} \ E!X \triangleright T\varphi \qquad (\textbf{Translation})$$

Let us now proceed to the reduction of truthmaking principles to grounding principles. Correia introduces theorems that express a reduction of truthmaking structural principles to grounding structural principles (see the Appendix). For instance, we can easily derive the following structural truthmaking principle (truthmaking factivity),

$$\text{If } X \models \varphi, \text{ then } \bigwedge_{\sigma \in E!X} \sigma \text{ and } T\varphi \qquad (\textbf{T-FACT})$$

from the corresponding structural grounding principle (grounding factivity),[4]

$$\text{If } \Delta \triangleright \varphi, \text{ then } \bigwedge_{\psi \in \Delta} \psi \text{ and } \varphi \qquad (\textbf{G-FACT})$$

[1] An issue that has already been widely discussed (Armstrong, 1997; Lewis, 2001; Horwich, 2008; Bigelow, 2009; Thomas, 2011; Blackburn, 2012; Asay, 2013, 2014, 2021, 2022; MacBride, 2013; Price et al., 2013; Simpson, 2019).

[2] Some exceptions can perhaps be found in the works of Hornsby (2005), Schnieder (2006), and Dodd (2007).

[3] See, for instance: Rodriguez-Pereyra, 2005, 2015; Schnieder, 2006; Schaffer, 2008, 2010b; Rosen, 2010; Correia et al., 2012, 2014; Fine, 2012a, 2012b, 2020; Tahko, 2013; Griffith, 2014; Cameron, 2016; Jago, 2018.

[4] These principles are introduced in (Correia, 2014).

Correia also derives some logical truthmaking principles from logical grounding principles. For instance, we can derive this truthmaking principle:

$$\text{If } X \models \varphi \text{ and } Y \models \Psi, \text{ then } X, Y \models \varphi \wedge \Psi \tag{\textbf{LT1}}$$

from this grounding principle

$$\text{If } T\varphi \text{ and } T\Psi, \text{ then } T\varphi, T\Psi \rhd T(\varphi \wedge \Psi) \tag{\textbf{LG1}}$$

and state the result in the following theorem:[5]

Theorem 1 (Correia, 2014)**.** (**LT1**) *follows from* (**LG1**).

Correia thus offers truthmaking and grounding principles concerning conjunction, disjunction, and the existential operator ($\exists$) along with the corresponding theorems that establish the reduction of the former to the latter.[6] However, at the end of his paper (2014), he points out that a complete reduction of truthmaking to grounding—a task he does not undertake—through a derivation of the former from the latter must take into account additional principles that he does not consider.

We will first offer some structural principles that Correia does not include but which can be found in much of the available literature (Schnieder, 2011; Fine, 2012a, 2012b; Poggiolesi, 2016):

$$\text{If } \Delta \rhd \varphi, \text{ then Not: } \Delta \rhd \neg\varphi \tag{\textbf{Non-contradiction}$_G$}$$

[7]

$$\text{If } \Delta_1 \rhd \varphi, \Delta_2 \rhd \varphi, ..., \Delta_n \rhd \varphi, \text{ then } \Delta_1, \Delta_2, ..., \Delta_n \rhd \varphi \tag{\textbf{Amalgamation}$_G$}$$

Similarly, in truthmaker theory, we usually encounter the following principles:

$$\text{If } X \models \varphi, \text{ then Not: } X \models \neg\varphi \tag{\textbf{Non-contradiction}$_T$}$$

$$\text{If } X_1 \models \varphi, X_2 \models \varphi, ..., X_n \models \varphi, \text{ then } X_1, X_2, ..., X_n \models \varphi \tag{\textbf{Amalgamation}$_T$}$$

Theorem 2. *Each truthmaking principle follows from the corresponding grounding principle.*

Proof. Each case follows by (**Translation**). ■

At this point, we add the structural rule of cut in grounding, which is important for the proofs of the following theorems:

$$\text{If } \Delta, \Psi \rhd \varphi \text{ and } \Lambda \rhd \Psi, \text{ then } \Delta, \Lambda \rhd \varphi \tag{\textbf{CUT}}$$

Concerning negation, the following principles are often accepted as the basis for this constant in grounding (Schnieder, 2011; Fine, 2012a, 2012b; Correia, 2013; Poggiolesi, 2016, 2018; Correia, 2017):

$$\text{If } T\varphi, \text{ then } T\varphi \rhd T\neg\neg\varphi \tag{\textbf{LG2}}$$

$$\text{If } T\neg\varphi, \text{ then } T\neg\varphi \rhd T\neg(\varphi \wedge \Psi) \tag{\textbf{LG3}}$$

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[5] Note that **Theorem**s 1–4 depend principally on (**Translation**) and (**G-FACT**).

[6] See the Appendix for the remaining theorems not considered in this section.

[7] This principle can be demonstrated from (**G-FACT**) and logic: assume $\Delta \rhd \varphi$ and, for reduction, suppose $\Delta \rhd \neg\varphi$. Hence, by (**G-FACT**), φ and $\neg\varphi$. Then, by logic, $\Delta \rhd \neg\varphi$ does not hold.

$$\text{If } T\neg\varphi \text{ and } T\neg\Psi, \text{ then } T\neg\varphi, T\neg\Psi \rhd T\neg(\varphi \vee \Psi) \quad \textbf{(LG4)}$$

In addition, we have the corresponding principles in truthmaker theory, which are also widely accepted (Mulligan et al., 1984; Pendlebury, 2010; Fine, 2012a; Asay, 2016; Fine, 2017; Tennant, 2017, 2018):

$$\text{If } X \models \varphi, \text{ then } X \models \neg\neg\varphi \quad \textbf{(LT2)}$$

$$\text{If } X \models \neg\varphi, \text{ then } X \models \neg(\varphi \wedge \Psi) \quad \textbf{(LT3)}$$

$$\text{If } X \models \neg\varphi \text{ and } Y \models \neg\Psi, \text{ then } X, Y \models \neg(\varphi \vee \Psi) \quad \textbf{(LT4)}$$

Accordingly, we can prove the following:

Theorem 3. *For $i = \{2, 3, 4\}$, **LTi** follows from **LGi**,* (**G-FACT**)*, and* (**CUT**)*.*

Proof. For (**LT2**), suppose that $X \models \varphi$. By (**Translation**), it follows that $E!X \rhd T\varphi$, and by (**G-FACT**), $T\varphi$. Now, by (**LG2**), $T\varphi \rhd T\neg\neg\varphi$ holds, and by (**CUT**), $E!X \rhd T\neg\neg\varphi$. Applying (**Translation**) again, we conclude that $X \models \neg\neg\varphi$.

For (**LT3**), suppose that $X \models \neg\varphi$. By (**Translation**), $E!X \rhd T\neg\varphi$ holds. Now, by (**G-FACT**), $T\neg\varphi$ obtains. By applying (**LG3**), $T\neg\varphi \rhd T\neg(\varphi \wedge \Psi)$ also holds, and then, by (**CUT**), $E!X \rhd T\neg(\varphi \wedge \Psi)$ holds as well. Finally, by (**Translation**) again, we conclude that $X \models \neg(\varphi \wedge \Psi)$.

For (**LT4**), suppose that $X \models \neg\varphi$ and that $Y \models \neg\Psi$. By (**Translation**), it holds that $E!X \rhd T\neg\varphi$ and that $E!Y \rhd T\neg\Psi$. By (**G-FACT**), it follows that $T\neg\varphi$ and $T\neg\Psi$. So now, by (**LG4**), $T\neg\varphi, T\neg\Psi \rhd T\neg(\varphi \vee \Psi)$ holds. Thus, by (**CUT**), $E!X, E!Y \rhd T\neg(\varphi \vee \Psi)$ also holds. Finally, we determine by (**Translation**) again that $X, Y \models \neg(\varphi \vee \Psi)$. ■

Concerning the universal ($\forall$), and following Fine (2012a), the language must be enriched by introducing a special totality predicate τ. This is not a standard predicate: "it is multi-grade, capable of receiving any number of terms [...] to make a formula" (Correia, 2013, p. 45). Hence, where $a, b, c, \ldots$ are constants of the language, the well-formed expression $\tau(a, b, c, \ldots)$ is to be understood as stating that "the objects denoted by $a, b, c, \ldots$ are all the objects that there are." Accordingly, the following are grounding principles for the universal quantifier:[8]

$$\begin{array}{r} \text{If } \varphi(a_1), \varphi(a_2), \ldots, \varphi(a_n) \text{ and } \tau(a_1, a_2, \ldots, a_n), \\ \text{then } \varphi(a_1), \varphi(a_2), \ldots, \varphi(a_n) \rhd \forall x\varphi(x) \end{array} \quad \textbf{(LG5)}$$

$$\begin{array}{r} \text{If } T\varphi(a_1), T\varphi(a_2), \ldots, T\varphi(a_n) \text{ and } \tau(a_1, a_2, \ldots, a_n), \\ \text{then } \forall x T\varphi(x) \rhd T\forall x\varphi(x) \end{array} \quad \textbf{(LG6)}$$

[8] The addition of $\tau(a, b, c, \ldots)$ to these principles has sparked a contentious debate concerning the ontological status of so-called "totality facts." They could be fusions of first-order states (Fine, 2012a), second-order states of affairs that genuinely exist in the spacetime world (Armstrong, 1997, 2004), linguistic artifacts with no necessary correlates in reality (Heil, 2003, 2006), empirical absences (Martin, 1996), or labeled proofs like Carnap's ω-rule (Tennant, 2017, 2018). In any case, whatever their exact nature, totality facts are necessary to ground universally quantified formulas. As Fine explains in (2012a, pp. 60-62), it is not sufficient for grounding the universal that φ holds for all constants; one must also assume that these constants denote all the objects of a given domain. That commitment is made explicit through the inclusion of the totality claim within the principle, as Fine observes in (2012a, p. 61). This entails the admittedly strong assumption that every object of the domain is named by a constant.

$$\text{If } T\varphi(a_1), T\varphi(a_2), ..., T\varphi(a_n) \text{ and } \tau(a_1, a_2, ..., a_n), \text{ then } T\varphi(a_1), T\varphi(a_2), ..., T\varphi(a_n) \rhd T\forall x\varphi(x) \tag{LG7}$$

Theorem 4. (**LG**7) *follows from* (**LG5**)*,* (**LG6**)*, and* (**CUT**)*.*

Proof. Suppose that $T\varphi(a_1), T\varphi(a_2), ..., T\varphi(a_n)$ and $\tau(a_1, a_2, ..., a_n)$. By (**LG6**), it holds that $\forall xT\varphi(x) \rhd T\forall x\varphi(x)$; and by (**LG5**), $T\varphi(a_1), T\varphi(a_2), ..., T\varphi(a_n) \rhd \forall xT\varphi(x)$. Then, by (**CUT**), we obtain that $T\varphi(a_1), T\varphi(a_2), ..., T\varphi(a_n) \rhd T\forall x\varphi(x)$, as desired. ■

The following is a principle that defines the universal in truthmaker theory (Pendlebury, 2010; Fine, 2017, 2020; Tennant, 2017, 2018):

$$\text{If } X \models \varphi(a_1), X \models \varphi(a_2), ..., X \models \varphi(a_n) \text{ and } \tau(a_1, a_2, ..., a_n), \text{ then } X \models \forall x\varphi(x) \tag{LT7}$$

We can now prove the following theorem:

Theorem 5. (**LT7**) *follows from* (**LG**7)*,* (**Translation**)*,* (**G-FACT**)*, and* (**CUT**)*.*

Proof. Suppose that $X \models \varphi(a_1), X \models \varphi(a_2), ..., X \models \varphi(a_n)$ and $\tau(a_1, a_2, ..., a_n)$. By (**Translation**), it follows that $E!X \rhd T\varphi(a_1), E!X \rhd T\varphi(a_2), ..., E!X \rhd T\varphi(a_n)$. By (**G-FACT**), we obtain that $T\varphi(a_1), T\varphi(a_2), ..., T\varphi(a_n)$. Then, by (**LG7**), the following holds: $T\varphi(a_1), T\varphi(a_2), ..., T\varphi(a_n) \rhd T\forall x\varphi(x)$. Finally, by (**CUT**), $E!X \rhd T\forall x\varphi(x)$; thus, by (**Translation**) again, $X \models \forall x\varphi(x)$, as desired. ■

Finally, we address two key principles extensively debated within truthmaker theory: the conjunction thesis and the disjunction thesis:

$$\text{If } X \models \varphi \wedge \psi, \text{ then } X \models \{\varphi, \psi\} \tag{CT}$$

$$\text{If } X \models \varphi \vee \psi, \text{ then } X \models \varphi \text{ or } X \models \psi \text{ or } X \models \{\varphi, \psi\} \tag{DT}$$

The following are two elimination rules that Fine provides in (2012a) (albeit in different notation) for conjunction and disjunction, respectively:

$$\text{If } \Delta \rhd T(\varphi \wedge \psi), \text{ then } \Delta \rhd \{T\varphi, T\psi\} \tag{Fine$_\wedge$}$$

$$\text{If } \Delta \rhd T(\varphi \vee \psi), \text{ then } \Delta \rhd T\varphi \text{ or } \Delta \rhd T\psi \text{ or } \Delta \rhd \{T\varphi, T\psi\} \tag{Fine$_\vee$}$$

Theorem 6. (**CT**) *and* (**DT**) *follow from* (**Fine**$_\wedge$) *and* (**Fine**$_\vee$)*, respectively.*

Proof. For (**CT**), let $X \models \varphi \wedge \psi$. By (**Translation**), it follows that $E!X \rhd T(\varphi \wedge \psi)$, and by (**Fine**$_\wedge$), $E!X \rhd \{T\varphi, T\psi\}$ holds. Now, by (**Translation**) again, $X \models \{\varphi, \psi\}$ holds as well.

For (**DT**), let $X \models \varphi \vee \psi$. By (**Translation**), it follows that $E!X \rhd T(\varphi \vee \psi)$, and by (**Fine**$_\vee$), $E!X \rhd T\varphi$ or $E!X \rhd T\psi$ or $E!X \rhd \{T\varphi, T\psi\}$. Finally, by (**Translation**) again, it obtains that $X \models \varphi$ or $X \models \psi$ or $X \models \{\varphi, \psi\}$. ■

2.2 Justifying the Reduction

A central thesis of this paper is that truthmaking can be reduced to grounding because the main truthmaking principles can be derived from basic grounding principles (Correia, 2013, 2014, 2017). Accordingly, a question that naturally arises is the following: Why should we regard this deduction as constituting a reduction of truthmaking to grounding? After all, the derivation could also hold in the reverse direction.

As previously noted, truthmaking is typically defined as stating that a truthbearer is true *in virtue of* the existence of a given entity (Armstrong, 1997, 2004; Rodriguez-Pereyra, 2005; Asay, 2022). That is, a truthbearer is true *because* some entity exists (Correia, 2014). Both formulations invoke terms like "because" and "in virtue of," which express metaphysical explanation or ontological dependence—concepts typically understood through the framework of "grounding." Without such notions, the explanatory role of truthmaking would be diminished. Consequently, the concept itself would lose coherence, since truthmaker theories aim to explain how the truth of truthbearers is explained by reference to specific entities (Asay, 2022).

Therefore, to make sense of the very notion of truthmaking, one must ultimately rely on the concept of grounding. Accordingly, fundamental principles can be established to govern the notion through which truthmaking has been defined—namely, grounding principles. Thus, there is an initial conceptual reason suggesting that the deduction involving grounding and truthmaking principles ought to proceed from grounding principles to truthmaking principles: the central tenet of truthmaking relies on concepts governed by grounding principles.

Another reason comes from logic. As currently formulated, grounding principles cannot be inferred from truthmaking principles. For example, (**LG1**) does not follow from (**LT1**), since from $T\varphi$ and $T\psi$, there is no clear route to derive $T\varphi, T\psi \rhd T(\varphi \wedge \psi)$ using (**LT1**). However, the question arises once more: Why not introduce auxiliary principles to derive (**LG1**) from (**LT1**), and more generally, to permit the systematic deduction of grounding principles from truthmaking principles?

It is worth emphasizing that the only auxiliary principle to which we are committed—one that articulates a connection between grounding and truthmaking—is (**Translation**). This principle is not arbitrary; rather, drawing on the preceding definition of "truthmaking," it articulates the standard view that truthmaking encompasses a dependence relation between entities and truthbearers. In other words, it posits that any truthmaking relation like $X \models \varphi$ can be reformulated as a grounding relation in which φ's truth is grounded in X's existence. However, (**Translation**) permits only grounding relations of this form to be translated back into truthmaking ones. For instance, $\Delta \rhd \varphi$ cannot be reformulated as a truthmaking relation via (**Translation**), because Δ need not involve existence claims about some entity, and φ may not fall under the predicate "being true."

In short, (**Translation**) permits any truthmaking relation to be recast as an instance of grounding, but not the reverse. This is the reason why this principle enables the derivation of all truthmaking principles from grounding principles. Indeed, the proofs of theorems in the previous section start with the antecedent of a truthmaking principle and, through the application of some grounding principle, derive its consequent. However, such an application is only available when performed on a grounding instance. Therefore, it is necessary to first translate the antecedent of the truthmaking principle into grounding instances using

(**Translation**).

Let us now consider the reverse process: deriving grounding principles from truthmaking ones. Analogously, one would prove theorems by assuming the antecedent of a grounding principle, then applying some truthmaking principle—provided such a principle exists, which is itself a significant assumption—to derive the consequent. However, this application can only be performed on an instance of truthmaking. Consequently, it would be necessary first to translate the antecedent of the grounding principle into some truthmaking instances. Therefore, at a minimum, we would need a principle similar to (**Translation**) that would allow us to move from any grounding relation to a corresponding truthmaking statement. If not, instances of grounding corresponding to the antecedent of a principle might remain untranslated into truthmaking relations, precluding adequate deduction from some truthmaking principle.

Nevertheless, introducing such a principle would require explaining how a relation confined to worldly entities and truthbearers could capture the full generality of grounding. Indeed, to move from any grounding relation to a truthmaking relation would imply that all ontological dependence relations—i.e., grounding instances—could be conceptualized as truthmaking relations, that is, as relations between entity names and truthbearers. Yet, there are clear cases of grounding that do not allow for this (see, for instance, the first five examples presented at the outset of §3). Consequently, such a principle would not only be less intuitive but would also risk appearing ad hoc, as it finds no support in the standard definitions of truthmaking and grounding.

Therefore, the direction of the deduction carried out in §2.1 is both appropriate and necessary, insofar as it is motivated by the very definition of truthmaking. By contrast, performing the deduction in the reverse direction would be inadequate. Once the deduction in §2.1 is completed, we demonstrate that truthmaking relations can be reconstructed as grounding relations, while preserving the logical structure of the former. From a deflationary perspective, if truthmaking and grounding principles fully capture the content of both notions, this implies that the concept of truthmaking is derived from that of grounding, thereby effecting what we refer to as a "reduction."

3. Weakening the World-to-Truth Thesis

In this section, it is argued that understanding truthmaking as grounding allows non-spatiotemporal entities (e.g., mathematical facts) to serve as truthmakers (§3.1). We then briefly rebut main objections to this argument (§3.2). Finally, we provide a brief clarification of the sense in which truthmaking constitutes a specific type of grounding (§3.3).

3.1 Extending the Domain of Truthmakers

Given the reduction of truthmaking to grounding in the preceding section, truthmaking can be seen as a kind of grounding. Therefore, restricting truthmaking to the postulation that worldly objects are truthmakers would be an additional constraint to which we need not commit ourselves, because grounding is not limited to taking worldly objects and propositions as its *relata*. Some proponents of grounding hold that a wide variety of different types of entities enter into grounding relations, and that one entity may ground another even if they belong to disparate ontological categories—see, for instance, Batchelor, 2010; Schaffer, 2010a; deRosset, 2013; Poggiolesi, 2016, 2018. Consider the following examples:

- One fact can ground another: the fact that snow is white grounds the fact that snow is either white or red.
- One object may ground another: Obama grounds his singleton.
- One property may ground another property: the property *being white* grounds *being white or square*.
- One object may ground a property: England grounds (in part) the property of *being queen of England*.
- One object may ground an event: Brutus grounds (in part) Brutus's stabbing of Caesar.
- One fact may ground a truth: the rose being red grounds the truth of "the rose is red."
- One truth may ground another: the truth of "the rose is red" grounds the truth of "the rose is red or the house is crowded."

Thus, grounding plausibly links a disparate assortment of entities in a wide variety of cases. As Batchelor argues (2010), any existential dependence relation can hold between entities of any category. This is because a relation of grounding can be conceptualized as a logical relation (in a broad sense of the term)—i.e., a relation defined in full by structural and logical principles (some of which are considered in §2 and in the Appendix). As such, there is nothing in it that imposes ontological constraints of any sort. Grounding principles could be envisaged that do so. However, such constraints would be established for independent theoretical reasons that go beyond what is commonly understood by "grounding" and thus do not need to be included as axioms or rules of any kind in a grounding logic, whose purpose is to capture formally the usual understanding of grounding (Correia, 2005, 2010, 2013, 2014, 2017, 2020, 2021; Schnieder, 2011; Fine, 2012a, 2012b; deRosset, 2013; Krämer et al., 2015; Poggiolesi, 2016, 2018, 2020; McSweeney, 2020).

Therefore, the grounding relation can be applied to any entity whatsoever because it does not impose ontological restrictions. Of course, this implies that, at first, any abstract object can be the ground of any ontological category. Hence, the grounding relation can be applied to mathematical objects, as they are, presumably, a kind of abstract object (Batchelor, 2010; Correia et al., 2012). We can allow, e.g., the mathematical fact *that seven plus five equals twelve* to be the ground for the truth of "$7 + 5 = 12$."

We now have compelling reasons to weaken the principle (**WT**). If grounding permits mathematical facts to serve as grounds for the truth of arithmetical statements, and if truthmaking is reducible to grounding, then (**WT**) should be revised to allow mathematical objects to function as legitimate truthmakers for such statements. In other words, given this reduction, truthmaking may be understood as a grounding relation. But grounding relations themselves do not impose ontological restrictions on the kinds of entities that may serve as *relata*. Therefore, there is no immediate or a priori reason to assume that truthmakers must belong to a specific ontological category—i.e., no ontological constraint is to be imposed, in principle, on the *relata* of the truthmaking relation.

Consider, for instance, the following truthmaking claim: $X \models 7 + 5 = 12$. According to the classical truthmaking approach, X must refer to a mereological whole with at least 12 spatiotemporal constituents (Bigelow, 1988; Armstrong, 2004; Cameron, 2008). However, by (**Translation**), we know that the above claim is the same as the following grounding statement: $E!X \rhd T(7+5=12)$, where $E!X$ can affirm the existence of an abstract mathematical object. Accordingly, since truthmaking reduces to grounding, X's ontological commitments align with those of $E!X$, eliminating the demand for spatiotemporal objects. Therefore, X can refer to a mathematical fact as an abstract object, allowing non-worldly entities to serve as truthmakers for arithmetical statements.

3.2 An Independent Argument

An objection might arise that reducing truthmaking to grounding is an unnecessary step, suggesting instead that weakening the principle (**WT**) is simpler. It suffices to point out that arithmetical statements do not appear to have truthmakers within the spacetime world, implying that (**WT**) is overly restrictive and should be adjusted to include other entities as truthmakers. In essence, it might be contended that (**WT**) should expand to include a broader range of entities as potential truthmakers, arguing that truthmaker theory could function effectively by merely positing extralinguistic truthmakers without adhering to (**WT**)'s stringent requirements.

However, this objection overlooks the foundational role of (**WT**) in truthmaker theory. Far from being an arbitrary constraint, (**WT**) is a core presupposition, supported by both historical and philosophical considerations. Historically, the pioneering work by Mulligan et al. (1984) already incorporates this principle, and Armstrong (1997, 2004) presents truthmaker theory as a contemporary articulation of correspondentist and realist theories of truth—admittedly, a perspective endorsed by the vast majority of modern truthmaker theorists (Rodriguez-Pereyra, 2005). Philosophically, the aim of anchoring truth in concrete spatiotemporal entities is to avoid a "floating" conception of truth and to establish an epistemically secure foundation. The concrete world, to which we presumptively have firm and secure epistemic access, is regarded as the most reliable basis (Bigelow, 1988, 2009).

The commitment to (**WT**) becomes evident in debates concerning the truthmakers for negated, counterfactual, arithmetical, or temporal statements. Absent (**WT**) constraints, one could readily posit entities such as negative facts, counterfactual entities, or abstract objects to resolve these issues expediently. The enduring nature of this debate highlights the foundational importance of (**WT**) within truthmaker theory, given the persistent difficulty in identifying concrete entities that can serve as truthmakers for such statements (Armstrong, 2004, 2007).

In light of these difficulties, it may appear reasonable to allow for extralinguistic truthmakers that are not constrained by spacetime. However, considering the foregoing discussion, this solution risks appearing ad hoc because it introduces such entities only reactively, that is, only when the canonical version fails in a particular problematic case, most notably in cases like arithmetical truths. Such a strategy lacks a general principle to determine when these entities should be posited, resulting in a theory that responds *ex post facto* to specific challenges. This undermines its unifying purpose and explanatory power because it resorts to bespoke metaphysical entities for each problematic case, such as for the truth of "$3+2=5$."

Essentially, proposing that the principle (**WT**) should encompass entities beyond spacetime simply because arithmetical statements (and similar statements) do not appear to involve spacetime objects as truthmakers, is akin to addressing a problem by invoking the very source of the issue—namely, the principle (**WT**)—but without the obstacles that cause the problem itself. However, what is really at stake is why those issues at the root of the problem should be addressed. The objection merely reframes the issue without offering substantive reasons for modifying (**WT**), proposing an alternative formulation that sidesteps difficulties but fails to explain why such a change is warranted.

By contrast, reducing truthmaking to grounding provides a principled and systematic non ad hoc alternative for weakening (**WT**). Grounding, as a relation spanning diverse ontological categories, offers a flexible yet coherent framework. Redefining truthmaking in these terms preserves the theory's core insight—that truth is rooted in a broad notion of reality—while accommodating statements with truthmakers beyond spacetime. For instance, arithmetical truths can be grounded in abstract entities like numbers, circumventing the need for an arbitrary expansion of (**WT**).

The now independent reason for correcting (**WT**) is that this principle can be reduced to a grounding relation, which inherently accommodates entities from any ontological category as *relata*. Thus, far from being unnecessary, the shift to grounding strengthens truthmaker theory. It addresses the limitations of (**WT**)—such as its handling of arithmetical statements—while preserving the theory's explanatory power. It also avoids the pitfalls of ad hoc adjustments, thereby ensuring a coherent justification for both "orthodox" and "non-orthodox" cases without resorting to piecemeal patches.

3.3 Truthmaking as a Restricted Kind of Grounding

The reduction of truthmaking to grounding offers independent justification for including non-spatiotemporal entities as *relata* in the truthmaking relation. In light of this, Simpson's (2019) insight that the explanatory roles of truthmaking and grounding appear increasingly similar gains particular relevance. However, by framing truthmaking as a form of grounding, this similarity risks collapsing into an identity between the two relations. If truthmaking is indeed reducible to grounding, there is no *prima facie* reason to consider truthmaking a relation that holds between truthbearers and objects. As a result, truthmaking has been effectively diluted into grounding.

These considerations suggest that if we define truthmaking as an unrestricted form of grounding, we should apply some constraint to at least one of its *relata*. Indeed, to call a kind of grounding "truthmaking" implies that at least one of its *relata* must be a linguistic object-—a truthbearer, something that can be made true—such as a sentence or a proposition. Accordingly, it is entirely legitimate to distinguish, among the different types of grounding relations, those in which what is grounded is a truth, and to call this type of grounding "truthmaking." The last two examples of grounding relationships at the beginning of §3.1 clearly illustrate truthmaking in this sense.

These ideas become clearer when formal tools are used. From the classical point of view, truthmaking is a dyadic relational predicate that can be extensionally defined by the set

$$\{\langle a, b\rangle : \ a \text{ is a worldly object and } b \text{ is a linguistic object}\} \qquad (\mathcal{T})$$

(provided we read truthmaking statements as "a makes b true"). If, as noted, grounding theorists do not impose restrictions on the types of *relata* used in grounding statements, then grounding could also be defined as

$$\{\langle a, b\rangle : \ a, b \text{ are objects}\} \qquad (\mathcal{G})$$

Given the reduction of truthmaking to grounding, it follows that $\mathcal{T}\subseteq\mathcal{G}$, and, therefore, that the set ($\mathcal{T}$) can be extended to cover all entities that ($\mathcal{G}$) already covers. However, given what we mean when we speak of truthmaking—i.e., an ontological dependence relation which must have a linguistic object as one of its *relata*—it must follow that $\mathcal{T}\subset\mathcal{G}$. This follows, in fact, because $\mathcal{G}\nsubseteq\mathcal{T}$ (see §2.2), and thus, $\mathcal{T}\not\equiv\mathcal{G}$, so it must be the case that $\mathcal{T}\subset\mathcal{G}$. Consequently, truthmaking could be extensionally defined as

$$\{\langle a, b\rangle : \ a \text{ is an object and } b \text{ is a linguistic object}\} \qquad (\mathcal{T}^*)$$

Defining truthmaking as ($\mathcal{T}^*$) is justified insofar as we want to identify truthmaking as a particular kind of grounding relation. Therefore, our reduction should only allow us to state that truthmaking is ontologically neutral as far as the domain of truthmakers it encompasses is concerned.

4. Deflating Truthmaker Theory

The approach to be developed in this section will be deflationary in the sense that the truthmaking relation need not be understood as requiring the existence of truthmakers at all. This implies that the truth of arithmetical statements can be explained without invoking any entities whatsoever.

The core of the argument is as follows. First, there are views on grounding that conceive of it without requiring it to be a relation involving *relata*. On the other hand, truthmaking can be understood as a particular kind of grounding, given the reduction proposed in §2. Consequently, since it is a kind of grounding, and assuming a conception of grounding that dispenses with *relata* (specifically, with grounds), there is no a priori reason to demand that truthmaking be construed as a relation that necessitates *relata*—specifically, truthmakers.

Accordingly, this section begins by examining specific deflationary approaches to grounding and subsequently uses them to argue for a deflationary conception of truthmaking. Let us also assume that the grounding relation "$\rhd$" is to be understood as meaning "because" (Merricks, 2007; Rosen, 2010; Schnieder, 2011; Fine, 2012a; Littland, 2015; Dasgupta, 2017).

Note also that grounding was previously understood as a dyadic relational predicate, but it will now be seen as a sentential operator. To grasp the difference, think of the former as defined by the set ($\mathcal{G}$) and the latter as a logical particle like "$\wedge$," "$\vee$," or "$\rightarrow$." As a predicate, grounding seems to require more ontological commitment than as a sentential operator (Poggiolesi, 2020), since ($\mathcal{G}$) consists of ordered pairs of objects. In contrast, as a sentential operator, grounding does not need to be defined by a set of specific entities. We will show that this distinction also applies to truthmaking.

4.1 Deflationary Grounding

Let us start with J. Melia (2005).[9] The main lesson we can draw from Melia's argument is that we can regard the particle "because" as implying nothing metaphysically substantive. Instead, if we take it as a primitive sentential operator (such as a primitive connective) rather than defining it in terms of a predicative relation, it is as ontologically innocent as other truth-functional operators, such as "$\wedge$," "$\vee$," or "$\rightarrow$." For just as we can use and understand sentences involving the familiar truth-functional connectives without considering statements that contain "$\wedge$," "$\vee$," or "$\rightarrow$" as genuine relations—i.e., without treating them as relating entities—and without treating the atomic sentences that may flank these operators as names for states of affairs, tropes, facts, events, or any other type of entity, so too can we understand "because" as a connective that is not something like a relation with *relata* (Melia, 2005, pp. 78-79).

Consequently, "because" can be understood as a primitive sentential operator rather than as a relation linking facts to facts, facts to propositions, objects to events, facts to sentences, and so on. Thus, from a principled or axiomatic point of view, by taking "$\rhd$" as a primitive operator in the grounding principles above, the set of such principles can be seen as a non-definitional description of grounding—i.e., one that does not require a reductive analysis of grounding into more primitive ontological dependence relations that demand specific categories of entities as *relata*. Instead, as a sentential operator, it connects sentences that, as with the classical connectives, do not have to be understood as names of objects.

Moreover, the fact that grounding principles exhaust this concept implies that the occurrences of "$\rhd$" in these principles do not reify grounding in reality in any substantive sense. Indeed, if grounding principles exhaust the content of the concept of grounding, then reductive analyses of grounding that define this notion by resorting to reality are simply wrong. Thus, grounding is not a substantive relation, because all the substantive content we can offer about grounding boils down to the enumeration of a plurality of principles that capture how "$\rhd$" works in different ways and different logical contexts. Strictly speaking, this indicates that there is a lack of commonality in the nature of the instances of "$\rhd$." Nothing is stated about their shared metaphysical or logical structure (Schnieder, 2011).

4.2 Deflationary Truthmaking

An argument analogous to that presented in §3.1 can now be offered to show that truthmaking does not require truthmakers, especially in the case of arithmetical truths. Since truthmaking can be reduced to grounding—and grounding itself can operate without any particular ground as *relata*—there is an independent reason to maintain that, insofar as truthmaking is understood as a form of grounding, it too can dispense with truthmakers.

[9] Melia (2005) presents his approach under the label of "truthmaking without truthmakers," and not exactly under that of grounding. Nevertheless, his argument can be extended by analogy to the "because" operator, i.e., to grounding, since he never establishes a difference between making a truth true and grounding its truth; at some points, he seems to identify both concepts. On the other hand, he also gives no substantive reasons for claiming that truthmaking does not need to engage with entities as truthmakers. Melia simply points out that this is a possible option. Still, this is open to criticism, as similarly argued in §3.2, since his argument may become ad hoc given that he uses this possibility to say against the ontological burden of truthmaking. It seems more appropriate, then, to start with "because" as a concept concerned with grounding in general, and to argue from there for a deflationary version of truthmaking, insofar as grounding is a more fundamental concept than truthmaking.

Indeed, if (**LT1**) can be derived from (**LG1**), and (**LG1**) does not entail substantive or inflationary ontological commitments to grounds, then, given that (**LG1**) holds a more foundational status than (**LT1**), this last principle is not obliged to incorporate additional ontological commitments to truthmakers beyond those already included in (**LG1**) as grounds. Therefore, in the same way as for grounding, and following Schnieder (2006, 2011), there is nothing beyond the logical and structural principles of truthmaking that is substantial or informative.

Everything substantive that can be established about truthmaking is reducible to principles that, insofar as they can be derived from ontologically empty principles, do not necessarily engage with any underlying metaphysical structure that imposes ontological constraints, or with any general truthmaking principle from which all others can be derived. The instances of "$\models$" provide all that is necessary to understand how the truth of statements is founded, and there is no commonality among them.

Does this approach still commit us to the actual existence of truthmakers? Following Melia (2005), since the notion of "truthmaking" falls under that of "grounding," "$\models$" can now be understood as a primitive sentential operator, that is, one that need not be defined in terms of a relation that links objects to propositions, and thus, that is not ontologically committed to them (McGrath, 2003; Hornsby, 2005; Merricks, 2007; Rychter, 2013; Tallant, 2017). In this sense, adopting Melia's reasoning, the sentences flanking "$\models$" need not be understood as names of objects of any kind, and especially not as names of truthmakers.

Let us look at this in more detail. Understanding truthmaking as we have so far, the following are examples of truthmaking instances: the state of affairs *a's being red* makes true the sentence "a is red"; the fact that *seven plus five equals twelve* makes true the sentence "$7 + 5 = 12$." In all these cases, the phrase "makes true" is understood as a relational predicate that takes names of truthmakers on one side and names of sentences on the other. Consequently, this interpretation entails an ontological commitment to such types of entities because being a relational predicate presupposes *relata*—that is, entities that must flank the predicate.

However, consider these same statements by taking "makes true" in these examples as a primitive sentential operator, i.e., as a non-truth-functional connective whose basic form could be "A *makes true* sentence S" (Melia, 2005). We may report the preceding cases of truthmaking as follows: a is red *makes true* the sentence "a is red," that seven plus five equals twelve *makes true* the sentence "$7 + 5 = 12$." Here, since "makes true" is conceptualized as a primitive sentential operator, there is no reason to believe that the sentences "a is red" and "seven plus five equals twelve" are names of any object. The fact that these sentences appear within statements containing operators gives us no *prima facie* reason to think that they must be names of some truthmaker. Thus, truthmaking need not be understood as a relationship between entities of any kind, but only as an operator that connects sentences. As a result, truthmaking can eliminate the need for ontological commitments with truthmakers.

4.3 Deflationary Truthmaking for Arithmetical Truths

Recall the question with which this paper began: What kind of entity—*if there is one*—explains the truth of an arithmetical statement? The deflationary perspective just outlined suggests that the truth of arithmetical statements can be explained without positing any entities as their truthmakers. We can explain whether instances of "$\models$" are obtained just by relying on truthmaking principles, without invoking objects of any kind.

Let us explain this last point concerning the instances of "$\models$" in arithmetical truths. We need some notation. Let PA^2 be the conjunction of the second-order Peano axioms. The only non-logical terms are $\mathbb{N}$, s, 0, $+$, and $\cdot$:

- $\mathbb{N}$ is the set of all natural numbers;
- 0 is a term for a constant;
- s is the successor function $s : \mathbb{N} \mapsto \mathbb{N}$ such that for all $n \in \mathbb{N}$, $s(n) = n + 1$;
- $+$ is a function $+ : \mathbb{N} \times \mathbb{N} \mapsto \mathbb{N}$ such that for all $m, n \in \mathbb{N}$, $+(m, 0) = m$ and $+(m, n + 1) = +(m, n) + 1$.
- $\cdot$ is a function $\cdot : \mathbb{N} \times \mathbb{N} \mapsto \mathbb{N}$ such that for all $m, n \in \mathbb{N}$, $m \cdot 0 = 0$ and $m \cdot (n+1) = m \cdot n + m$.

Denote by $PA^2(\mathbb{N}, s, 0, +, \cdot)$ the non-logical terms used in the axioms. According to the recursive definition provided for the addition function, it can be demonstrated that $+[s(s(0)), s(s(s(0)))] = s(s(s(s(s(0)))))$ as follows:

$$
\begin{aligned}
&+(s(s(0)), s(s(s(0)))) = \\
&s(+(s(s(0)), s(s(0)))) = \\
&s(s(+(s(s(0)), s(0)))) = \\
&s(s(s(+(s(s(0)), 0)))) = \\
&s(s(s(s(s(0)))))
\end{aligned}
$$

The expression "$+[s(s(0)), s(s(s(0)))] = s(s(s(s(s(0)))))$" will be denoted by Δ. This formula is part of Peano arithmetic system, in which natural numbers are recursively constructed from 0 using the successor function. Accordingly, the preceding demonstration establishes, within this framework, that Δ is derived from how the Peano structure defines the successor and addition functions. We shall refer to the assertion that Δ is obtained from the definitions of the addition and successor functions in Peano arithmetic as $\Delta(\mathbb{N}, s, 0, +, \cdot)$.

Certainly, "$+[s(s(0)), s(s(s(0)))] = s(s(s(s(s(0)))))$" merely constitutes a base-1 notational variant of "$2 + 3 = 5$." However, this formal expression elucidates the underlying structure of "$2 + 3 = 5$" in Peano arithmetic. Therefore, the notation $\Delta(\mathbb{N}, s, 0, +, \cdot)$ encodes that the identity statement "$2 + 3 = 5$" is similarly founded on the structure of the natural numbers expressed by the Peano axioms and, specifically, on how this structure defines the non-logical terms contained in such a statement. This motivates a grounding principle that reflects this dependence relation: if $T[\Delta(\mathbb{N}, s, 0, +, \cdot)]$, then $T[\Delta(\mathbb{N}, s, 0, +, \cdot)] \rhd T(2 + 3 = 5)$. That is, granting the way "$+$" is defined in Peano arithmetic, it follows that if $\Delta(\mathbb{N}, s, 0, +, \cdot)$ holds in that structure—i.e., if the assertion that Δ is derived from the definition of "$+$" is itself satisfied or true within Peano arithmetic—then, that $\Delta(\mathbb{N}, s, 0, +, \cdot)$ holds true grounds the truth of "$2 + 3 = 5$." Naturally, the antecedent of the principle is met when Δ is indeed derivable within PA^2.

Generalizing this point, for any naïve arithmetical statement α (such as "$2 + 3 = 5$"), and for any formalization Λ of this statement in the structure of Peano arithmetic, it follows that

$$\text{If } T[\Lambda(\mathbb{N}, s, 0, +, \cdot)], \text{ then } T[\Lambda(\mathbb{N}, s, 0, +, \cdot)] \rhd T\alpha \qquad (\rhd\textbf{PA})$$

In §4.1, it was argued that grounding principles need not be understood as invoking substantive entities as grounds. Accordingly, since ($\rhd$**PA**) is a grounding principle, there is no a priori reason to maintain that "$\triangleright$," as it appears in this principle, relates entities of any sort; consequently, one should not necessarily infer that ($\triangleright$**PA**) entails a commitment to mathematical entities as grounds.

Mathematical structuralism also supports the view that ($\triangleright$**PA**) does not motivate mathematical objects as grounds. Observe that ($\triangleright$**PA**) takes as grounds the truth of the statement that Λ is derived from how the Peano structure defines the non-logical terms occurring in Λ. In this sense, ($\triangleright$**PA**) may be understood as a structuralist principle, insofar as the ultimate foundation of the truth it grounds is identified with the Peano structure itself—that is, with a network of axiomatic definitions rather than with individual objects.

To elaborate, according to structuralism, mathematical theories like PA^2 do not describe numbers as objects distinguished by any intrinsic or essential property. Instead, mathematical objects should be treated as mere placeholders in functional relations, lacking specific properties (Benacerraf, 1965; Hellman, 1991, 1998, 2018; Reck et al., 2000; Florio, 2018). Accordingly, what is ontologically relevant are the relations between the pertinent items—namely, the relations between the elements of $\mathbb{N}$ and the functions s, $\cdot$, and $+$—rather than any intrinsic nature they may have, which is disregarded.

Therefore, by grounding arithmetical truth in a structure, ($\triangleright$**PA**)—when viewed through a structuralist lens—shows that one need not invoke individual mathematical objects as grounds. As argued by structuralism, the commitment to structures does not imply a commitment to particular entities, such as numbers.[10] Thus, structuralism reinforces the argument that arithmetical truths can be explained without committing to mathematical objects with intrinsic properties.

In addition, assuming in ($\triangleright$**PA**) that the assertion $\Lambda(\mathbb{N}, s, 0, +, \cdot)$ holds true involves minimal ontological commitment: it merely necessitates the existence of a proof of Λ within PA^2. Thus, the sole requirement stipulated by the antecedent of ($\triangleright$**PA**) is precisely that such a proof exists in PA^2. But the syntactic nature of demonstrations implies that the existence of a proof of Λ in PA^2 is a purely formal matter, based on the manipulation of symbols according to the inference rules and axioms of PA^2, without presupposing the existence of complex semantic constructs such as models. Furthermore, since proofs are primarily epistemic entities, they do not represent a substantive ontological commitment.

Inspired by ($\triangleright$**PA**), we can motivate a principle that expresses truthmaking in the context of Peano arithmetic—that is, a principle expressing how arithmetical statements are rendered true within this system. As before, let α denote a naïve arithmetical statement, and let Λ be its formalization in Peano arithmetic. As in ($\triangleright$**PA**), $\Lambda(\mathbb{N}, s, 0, +, \cdot)$ stands for the assertion that Λ is derivable from Peano's axioms. Accordingly, the following principle may reasonably be regarded as admissible:

$$\text{If } PA^2(\mathbb{N}, s, 0, +, \cdot) \models \Lambda(\mathbb{N}, s, 0, +, \cdot), \text{ then } PA^2(\mathbb{N}, s, 0, +, \cdot) \models \alpha \qquad (\models\textbf{PA})$$

Note that, according to the principle, the statement $\Lambda(\mathbb{N}, s, 0, +, \cdot)$ is true when the definitions of the non-logical terms appearing in Λ, as provided by PA^2, allow for the derivation of Λ. This claim is plausible because it is the non-logical terms as defined in PA^2 that imply the derivability of Λ within PA^2, and hence the truth of the statement: "Λ is derivable from the relevant definitions in PA^2." This justifies the antecedent of ($\models$**PA**). For if

[10] The fact that there is no ontological commitment to individual objects is still not supposed to require reifying the underlying structures (Resnik, 1997). The commitment to structures as abstract objects is likewise rejected by certain proponents of structuralism (Hellman, 1996).

it is indeed the case that the definitions of the non-logical terms permit a derivation of Λ, then $\Lambda(\mathbb{N}, s, 0, +, \cdot)$ is true in virtue of the non-logical terms as defined in PA^2—that is, $PA^2(\mathbb{N}, s, 0, +, \cdot) \models \Lambda(\mathbb{N}, s, 0, +, \cdot)$.

Concerning the reduction of truthmaking to grounding, the following theorem can be established:

Theorem 7. ($\models$**PA**) *follows from* ($\rhd$**PA**)*.*

Proof. Suppose that $PA^2(\mathbb{N}, s, 0, +, \cdot) \models \Lambda(\mathbb{N}, s, 0, +, \cdot)$ is the case. By (**Translation**), it follows that $E!PA^2(\mathbb{N}, s, 0, +, \cdot) \rhd T[\Lambda(\mathbb{N}, s, 0, +, \cdot)]$, and by (**G-FACT**), it holds that $T[\Lambda(\mathbb{N}, s, 0, +, \cdot)]$. By ($\rhd$**PA**), $T[\Lambda(\mathbb{N}, s, 0, +, \cdot)] \rhd T\alpha$ obtains, and by (**CUT**), $E!PA^2(\mathbb{N}, s, 0, +, \cdot) \rhd T\alpha$ holds as well. By (**Translation**) again, we finally deduce that $PA^2(\mathbb{N}, s, 0, +, \cdot) \models \alpha$, as desired. ■

This theorem proves that ($\models$**PA**) is reducible to ($\rhd$**PA**), and since this last principle does not necessarily imply a substantive grounding relation, neither does ($\models$**PA**) concerning truthmaking. Accordingly, we have independent reasons to affirm that ($\models$**PA**) carries no ontological commitment to objects as truthmakers—specifically, to numbers as the truthmakers of arithmetical statements—given that ($\rhd$**PA**) does not require numbers to serve as grounds. That is, by treating "$\models$" as a sentential operator, ($\models$**PA**) need not be understood as relating entities of any kind. Therefore, the explanation of the truth of an arithmetical statement such as "$3 + 2 = 5$" in terms of truthmaking is not one that necessarily engages with truthmakers, and in particular, with numbers as truthmakers. It is in this precise sense that the truth of arithmetical statements can be explained without invoking substantive ontological commitments.[11]

Additionally, ($\models$**PA**) does not have to be imposed as the sole principle for explaining arithmetical truth in terms of truthmaking. For instance, neo-Fregeans could resort to Hume's principle to ground arithmetical truths. For other examples, the reader may wish to refer to (Shumener, 2017, 2020). The discussion now focuses on which of these truthmaking principles best captures what is required to explain the truth of a statement such as "$7 + 5 = 12$":[12] Is it better to stick to some structure to explain arithmetical truth, or is it

[11] This assertion does not entail that truthmaking principles, such as ($\models$**PA**), themselves explain arithmetical truths. Should the truthmaking principles developed in the context of PA^2 suffice to explain arithmetical truth, it would follow—via the reduction of truthmaking to grounding—that a proof of Peano arithmetic's consistency could be obtained from various grounding principles, contingent upon their consistency. However, ($\rhd$**PA**) and ($\models$**PA**) do not serve as explanations of arithmetical truth; rather, they delineate how arithmetical statements are grounded or rendered true within systems such as PA^2, PA^1, and analogous frameworks. The basis of arithmetical truth might reside in a variety of theoretical constructs, including PA^2, PA^1, Robinson's arithmetic, Fregean principles, or comparable systems, but not in such principles themselves. The reason is that grounding and truthmaking principles do not form part of these arithmetic systems. Instead, at a metatheoretical level, they elucidate how arithmetical statements are rendered true within such systems.

[12] Importantly, ($\rhd$**PA**) and ($\models$**PA**) do not constitute a method for proving arithmetical statements within a formal system. Their antecedents presuppose that the relevant arithmetical statement has already been derived in the system. Hence, referring back to footnote 11, invoking them in an attempt to prove something like $Cons(PA)$ would require an antecedent proof of $Cons(PA)$ within PA itself—a result explicitly precluded by Gödel's second incompleteness theorem. Yet, the very suggestion of deriving $Cons(PA)$ from these principles oversteps because truthmaking and grounding principles do not belong to Peano's formal system. Offering an ontological account of what renders an arithmetical sentence true is a metatheoretical pursuit, extrinsic to PA. These principles are not designed to provide a method for deriving arithmetical statements within the object language of PA. Instead, they fulfill a normative role, illuminating the "make true" relation within some arithmetic domain, without constituting a self-contained axiomatic system capable of yielding theorems within PA.

more appropriate to resort to Humean principles?

Nonetheless, one might suggest, in a manner analogous to the objection raised in §3.2, that since arithmetical truths do not appear to be about the world, appealing to grounding becomes superfluous. Hence, it could be maintained directly that arithmetical truths lack truthmakers. The objection might go even further, claiming that arithmetical truths are not about any entity—not even abstract ones—and concluding, without recourse to grounding, that arithmetical truths require no truthmakers whatsoever. The objection targets principle (**WT**) directly, calling for its complete abandonment. This rejection is motivated by the existence of statements that do not seem to be about any object.

Since this objection is likewise a critique of (**WT**) prompted by particular cases (such as arithmetical statements), it can be addressed employing an argument analogous to that offered in §3.2. Indeed, the motivation invoked at the end of the previous paragraph is merely ad hoc. It does not provide any independent reasons for dispensing with truthmakers in the case of arithmetical truths.

Truthmaker theory is motivated by the insight that truth is anchored in reality. Therefore, to assert from within the theory itself that certain truths lack truthmakers—simply because they do not seem to invoke any object in the explanation of their truth—is merely a post hoc response to specific problems that emerge when the canonical version of the theory fails to yield the desired results. In such cases, no independent criterion is offered for why truthmakers should be dispensed with in some instances, thereby undermining the unifying and explanatory power of truthmaker theory. That is to say, without recourse to more fundamental notions than truthmaking to develop a truthmaking approach that dispenses with truthmakers, we run the risk of our argument being ad hoc.

Accordingly, reducing truthmaking to grounding does offer an independent and theoretically motivated basis for dispensing with truthmakers. If grounding can be conceived as a relation without *relata*, and if truthmaking is a kind of grounding relation (given the effected reduction), then we have independent reasons—that is, reasons external to the theory itself—for asserting that truthmaking, as a type of grounding, can similarly forgo *relata*—specifically, truthmakers for arithmetical truths.

5. *Do We Still Need Mathematical Entities?*

We argued that a deflationary approach to arithmetical truth does not require objects as truthmakers. Yet, defending a deflationary view of "$\models$" without postulating facts, concrete or abstract entities as truthmakers, differs from claiming no reasons exist to believe in such entities. One might accept a deflationary account of "$\models$" without truthmaker entities but still posit numbers to explain why accepting PA^2 over other arithmetical structures. However, the issue concerning the truth of arithmetical statements, as posed by truthmaker theory, is whether there are truthmakers that explain the truth of such statements. Numbers are indeed candidates for truthmakers, but the truth of these statements depends on whether numbers, if they exist, can form appropriate truthmaking relations with them, not on their actual existence.

From the perspective of truthmaker theory, numbers can serve as truthmakers for arithmetical statements, suggesting their existence is required for the statements' truth. Nonetheless, adopting "$\models$" as a primitive sentential operator allows the truth of arithmetical statements to be explained without necessitating numbers or other entities as truthmakers, meaning mathematical entities are not required to enter truthmaking relations. Whether it is metaphysically advantageous for the philosophy of mathematics to include numbers in our ontology remains a different question.

For instance, the incorporation of numbers within our ontology may facilitate the acceptance of specific truthmaking principles over others. It is in this sense that the deflationary truthmaking approach to arithmetical truth proposed in this paper does not necessarily engage with entities of any kind as truthmakers, but at the same time legitimizes, or at least does not rule out, a debate on the existence of such entities. According to this approach, it is reasonable to argue for the existence of, e.g., numbers as abstract objects of some kind without accepting them as truthmakers of arithmetical statements.

A distinct question is: what ontological commitments does this deflationary approach entail? Some have already surfaced in the previous section. For instance, in theorem 7, the formula $E!PA^2(\mathbb{N}, s, 0, +, \cdot) \rhd T[\Lambda(\mathbb{N}, s, 0, +, \cdot)]$ is displayed in the proof. Since $PA^2(\mathbb{N}, s, 0, +, \cdot)$ denotes the non-logical terms that appear in the Peano axioms, it is quite natural to read "$E!PA^2(\mathbb{N}, s, 0, +, \cdot)$" as the statement that affirms the existence of these terms, i.e., the existence of the set $\mathbb{N}$ and the functions s, $\cdot$, and $+$.

According to this existential assertion, we seem to acknowledge abstract entities, such as sets. Therefore, on the one hand, numbers might be *useful* for justifying our commitments with truthmaking principles, and on the other hand, sets are *necessary* for obtaining the corresponding reductive theorem. Hence, our deflationary approach must engage with, at the very least, sets as abstract entities of some sort.

Nonetheless, the fact that we use set theory in our metalanguage to derive ($\models$**PA**) from ($\rhd$**PA**) does not mean that truthmaking principles themselves are committed to sets *qua* truthmakers. For this to be the case, sets would have to enter into explanatory truth relations, expressed by "$\models$," with arithmetical statements. But there is nothing in ($\models$**PA**) or the corresponding theorems and proofs that suggests such a move. This is all the truer if "$\models$" is understood in this principle as a primitive sentential operator. Therefore, just as numbers are useful, but not necessary as truthmakers, sets are required, but not as truthmakers.

Some significant consequences follow from these considerations. First, the deflationary proposal of this paper is ontologically more flexible than structuralism. As argued in this section, we neither reject the existence of sets or numbers—as Hellman (1996) does—nor deny that mathematical objects may possess intrinsic properties—as Benacerraf (1965) argues. What is denied, instead, is that these objects serve as truthmakers.[13] The detour to structuralism only serves to reinforce this argument. Since some grounding principles—such as ($\rhd$**PA**)—ultimately resort to a network of axiomatic definitions as grounds, arithmetical truths are explained purely in terms of structural relations. However, as the structuralist emphasizes, the commitment to structures does not imply a commitment to individual objects.

Finally, the analysis performed in this last section reveals that there is a connection with ontology: sets are necessary to explain why ($\models$**PA**) follows from ($\rhd$**PA**), and thus to explain

[13] Another significant methodological distinction between the present proposal and structuralism lies in their respective aims. Structuralism primarily concerns itself with the ontological status of mathematical entities, redefining them as positions within a given structure. In contrast, the argument developed in §4 seeks to clarify how arithmetical statements attain their truth without resorting to specific truthmakers. In this sense, while structuralism asks, "What is the number 2?" and answers that it is a placeholder in the Peano structure, our approach inquires, "Why is the statement '$2 + 2 = 4$' true?" and responds by appealing to a structuralist insight: its truth is grounded in a coherent network of axiomatic definitions rather than in individual mathematical objects.

why ($\models$**PA**) does not contain ontological commitments to truthmakers for explaining the truth of arithmetical statements. Hence, a commitment to sets arises from a principle that grounds arithmetical truth. In a quite precise sense, arithmetical truth reveals some ontological commitments through a procedure that allows us to conclude that truthmakers are not required to make arithmetical statements true. Consequently, our theory of truthmaking without truthmakers uncovers ontological commitments by considering arithmetical truth, but without committing to genuine truthmakers. Thus, if (**WT**) is interpreted as the belief that truth somehow reveals what exists, using arithmetic as an example, the suggested principled approach to truthmaking satisfies that principle in this specific sense. This is a clear advantage over other approaches that dispense with truthmakers (see, e.g., (Hornsby, 2005) and (Melia, 2005)) because (arithmetical) truth remains rooted in reality.

6. Conclusion

The central argument of this paper can be articulated as follows:

(a) Grounding accommodates (a_1) a wide variety of entities as *relata* and (a_2) in certain deflationary conceptions, no *relata* at all.
(b) Truthmaking can be reduced to grounding, as explained in §2.
(c) Therefore, truthmaking can be conceptualized as a specific form of grounding.
(d) Consequently, there are independent (non ad hoc) reasons to assert that, based on (a_1), truthmaking can incorporate diverse non-spatiotemporal entities as truthmakers—particularly for arithmetical truths—(R_1) and, based on (a_2), it can eliminate the need for truthmakers—especially in the case of arithmetical truths (R_2).

The conclusions (R_1) and (R_2) are grounded in rationales distinct from those of truthmaking. As a result, they offer two independent and different responses to the question posed at the outset of this paper: *What kind of entity*—if there is one—*explains the truth of an arithmetical statement?* Accordingly, they represent two divergent philosophical perspectives on the explanation of arithmetical truth.

Of the responses, the second (R_2) is arguably the most striking. Contrary to what inflationary metaphysical positions, or even referential semantics, put forward, there is no need to posit any entity, whether worldly or abstract, to explain the truth of arithmetical statements. All that is needed is to define a set of truthmaking principles that capture the structure of the arithmetical statements whose truth is to be explained. However, these principles do not require ontologically substantive entities as truthmakers, as we have demonstrated by resorting to a deflationary account of grounding.

It should be noted, however, that this deflationary perspective may conflict with the approach developed in §3, which allows for any entity to be a truthmaker. In this proposal, it is permissible (but, importantly, not required) to claim, for instance, that Platonic entities can be the truthmakers of arithmetical truths, which is not consistent with ontological deflationism. Thus, the point of this paper is not to defend one account over the other, but rather to demonstrate that both options can be derived from grounding, and that this allows for an argument for both that is not ad hoc.

Appendix: Other Truthmaking and Grounding Principles

In this appendix, we examine the other grounding and truthmaking principles that Correia discusses in (2014). Since he does not provide proofs of the corresponding theorems, we attempt to deliver them ourselves. The appendix also covers certain principles not included by Correia.

Structural Principles

For grounding:

$$\text{If } \Delta \rhd \varphi, \text{ then } \bigwedge_{\psi \in \Delta} \psi \text{ and } \varphi \qquad \textbf{(G-FACT)}$$

$$\text{If } \Delta \rhd \varphi, \text{ then } \Box(\Delta \to \Delta \rhd \varphi) \qquad \textbf{(G-RIG)}$$

$$\text{If } \Delta \rhd \varphi, \text{ then } \Box(\Delta \to \varphi) \qquad \textbf{(G-NEC)}$$

$$\text{If } \Delta, \text{ then Not: } \Delta \rhd \Delta \qquad (\textbf{Irreflexivity}_G)$$

For truthmaking:

$$\text{If } X \models \varphi, \text{ then } \bigwedge_{\sigma \in E!X} \sigma \text{ and } T\varphi \qquad \textbf{(T-FACT)}$$

$$\text{If } X \models \varphi, \text{ then } \Box(E!X \to X \models \varphi) \qquad \textbf{(T-RIG)}$$

$$\text{If } X \models \varphi, \text{ then } \Box(E!X \to T\varphi) \qquad \textbf{(T-NEC)}$$

$$\text{If } X, \text{ then Not: } X \models X \qquad (\textbf{Irreflexivity}_T)$$

Theorem 8. *Each structural truthmaking principle follows from each structural grounding principle.*

Proof. For (**T-FACT**). Let $X \models \varphi$. By (**Translation**), it follows that $E!X \rhd T\varphi$, and by (**G-FACT**), $\bigwedge_{\psi \in E!X} \psi$ and $T\varphi$ obtain.

For (**T-RIG**). Let $X \models \varphi$. By (**Translation**), $E!X \rhd T\varphi$ holds. By (**G-RIG**), $\Box(E!X \to E!X \rhd T\varphi)$ holds as well. Finally, by (**Translation**), it follows that $\Box(E!X \to X \models \varphi)$.

For (**T-NEC**). Let $X \models \varphi$. By (**Translation**), $E!X \rhd T\varphi$ obtains, and by (**G-NEC**), $\Box(E!X \to T\varphi)$ holds.

For (**Irreflexivity**$_T$), suppose that X is the case. By Tarski's biconditional ($\varphi \leftrightarrow T\varphi$), it follows that TX. Now, by (**Irreflexivity**$_G$), Not: $TX \rhd TX$ holds, and by (**Translation**), Not: $X \models X$ holds as well. ■

Logical Principles

For truthmaking:

$$\text{If } X \models \varphi \text{ and } Y \models \Psi, \text{ then } X, Y \models \varphi \wedge \Psi \qquad \textbf{(LT1)}$$

$$\text{If } X \models \varphi \text{ or } X \models \Psi, \text{ then } X \models \varphi \vee \Psi \qquad \textbf{(LT2*)}$$

$$\text{If } X \models f(a), \text{ then } X \models \exists x f(x) \qquad \textbf{(LT3*)}$$

For grounding:

$$\text{If } \varphi \text{ and } \Psi, \text{ then } \varphi, \Psi \rhd \varphi \wedge \psi \tag{LG1*}$$

$$\text{If } \varphi, \text{ then } \varphi \rhd \varphi \vee \Psi \tag{LG2*}$$

$$\text{If } f(a), \text{ then } f(a) \rhd \exists x f(x) \tag{LG3*}$$

$$\text{If } T\varphi \text{ and } T\Psi, \text{ then } T\varphi \wedge T\Psi \rhd T(\varphi \wedge \Psi) \tag{SG1}$$

$$\text{If } T\varphi \text{ or } T\Psi, \text{ then } T\varphi \vee T\Psi \rhd T(\varphi \vee \Psi) \tag{SG2}$$

$$\text{If } Tf(a), \text{ then } \exists x Tf(x) \rhd T\exists x f(x) \tag{SG3}$$

$$\text{If } T\varphi \text{ and } T\Psi, \text{ then } T\varphi, T\Psi \rhd T(\varphi \wedge \Psi) \tag{WG1}$$

$$\text{If } T\varphi, \text{ then } T\varphi \rhd T(\varphi \vee \Psi) \tag{WG2}$$

$$\text{If } Tf(a), \text{ then } Tf(a) \rhd T\exists x f(x) \tag{WG3}$$

Theorem 9. *For $i \in \{1, 2, 3\}$, **WGi** follows from **SGi**, **LGi**, and* (**CUT**).

Proof. For (**WG1**). Let $T\varphi$ and $T\Psi$. By (**SG1**), it follows that $T\varphi \wedge T\Psi \rhd T(\varphi \wedge \Psi)$, and by (**LG1***), $T\varphi, T\Psi \rhd T\varphi \wedge T\Psi$ holds. By (**CUT**), it finally holds that $T\varphi, T\Psi \rhd T(\varphi \wedge \Psi)$.

For (**WG2**). Let $T\varphi$. Then, by (**SG2**), $T\varphi \vee T\Psi \rhd T(\varphi \vee \Psi)$ obtains. By (**LG2***), $T\varphi \rhd T\varphi \vee T\Psi$ holds as well. Now, by (**CUT**), $T\varphi \rhd T(\varphi \vee \Psi)$ finally follows.

For (**WG3**). Let $Tf(a)$. By (**SG3**), $\exists x Tf(x) \rhd T\exists x f(x)$ holds, and by (**LG3***), $Tf(a) \rhd \exists x Tf(x)$ obtains. Finally, by (**CUT**), $Tf(a) \rhd T\exists x f(x)$ holds as well. ■

Theorem 10. *For $i \in \{1, 2, 3\}$, **LTi** follows from **WGi**,* (**G-FACT**)*,* (**CUT**)*, and* (**Translation**)*.*

Proof. For (**LT1**). Let $X \models \varphi$ and $Y \models \Psi$. By (**Translation**), it follows that $E!X \rhd T\varphi$ and $E!Y \rhd T\Psi$. Now, by (**G-FACT**), $T\varphi$ and $T\Psi$ hold. By (**WG1**), $T\varphi, T\Psi \rhd T(\varphi \wedge \Psi)$ obtains. So now, if $E!X \rhd T\varphi$, $E!Y \rhd T\Psi$ and $T\varphi, T\Psi \rhd T(\varphi \wedge \Psi)$, it holds by (**CUT**) that $E!X, E!Y \rhd T(\varphi \wedge \Psi)$, and then, by (**Translation**), that $X, Y \models \varphi \wedge \Psi$.

For (**LT2***). Let $X \models \varphi$. By (**Translation**), $E!X \rhd T\varphi$ obtains, and by (**G-FACT**), $T\varphi$ holds. Now, by (**WG2**), $T\varphi \rhd T(\varphi \vee \Psi)$ obtains as well. So, by (**CUT**), it follows that $E!X \rhd T(\varphi \vee \Psi)$, and by (**Translation**), $X \models \varphi \vee \Psi$ holds as well, as desired.

For (**LT3***). Let $X \models f(a)$. By (**Translation**), $E!X \rhd Tf(a)$ holds, and by (**G-FACT**), $Tf(a)$. So now, by (**WG3**), it follows from $Tf(a)$ that $Tf(a) \rhd T\exists x f(x)$. By (**CUT**), it then holds that $E!X \rhd T\exists x f(x)$, and so, by (**Translation**), $X \models \exists x f(x)$ holds as well. ■

Conceptualism and Necessitation

Consider the following further principles:

$$\text{If } \Delta \rhd \varphi, \text{ then } T\Delta \rhd T\varphi^{14} \qquad \textbf{(SG4)}$$

$$\text{If } T\varphi, \text{ then } \varphi \qquad (\textit{Tarski}^{\rightarrow})$$

Theorem 11. *For $i \in \{1, 2, 3\}$, **WGi** follows from **LGi**,* (**SG4**)*, and* (*Tarski*$^{\rightarrow}$).

Proof. For (**WG1**). Let $T\varphi$ and $T\Psi$. By (*Tarski*$^{\rightarrow}$), it follows that φ and Ψ, and given this, by (**LG1**), $\varphi, \Psi \rhd \varphi \wedge \Psi$ holds. Finally, by (**SG4**), $T\varphi, T\Psi \rhd T(\varphi \wedge \Psi)$ holds as well, as desired.

For (**WG2**). Let $T\varphi$. By (*Tarski*$^{\rightarrow}$), it follows that φ, and so, by (**LG2***), $\varphi \rhd \varphi \vee \Psi$. But then, by (**SG4**), $T\varphi \rhd T(\varphi \vee \Psi)$ follows.

For (**WG3**). Let $Tf(a)$. As before, by (*Tarski*$^{\rightarrow}$), it follows that $f(a)$, and by (**LG3***), that $f(a) \rhd \exists x f(x)$. But then, by (**SG4**), $Tf(a) \rhd T\exists x f(x)$ holds. ■

Theorem 12. *If $X \models \varphi$ and $\varphi \rhd \Psi$, then $X \models \Psi$.*

Proof. Suppose that $X \models \varphi$ and $\varphi \rhd \Psi$. By (**Translation**), $E!X \rhd T\varphi$ holds, and by (**SG4**), $T\varphi \rhd T\Psi$ holds as well. By (**CUT**), it follows that $E!X \rhd T\Psi$, and by (**Translation**), that $X \models \Psi$. ■

Neutral Logical Principles for Grounding

$$\text{If } \Delta \rhd \varphi \text{ and } \Lambda \rhd \Psi, \text{ then } \Delta, \Lambda \rhd \varphi \wedge \psi \qquad \textbf{(LG1+)}$$

$$\text{If } \Delta, \Psi \wedge \sigma \rhd \varphi, \text{ then } \Delta, \Psi, \sigma \rhd \varphi \qquad \textbf{(LG1-)}$$

$$\text{If } \Delta \rhd \varphi \text{ or } \Delta \rhd \Psi, \text{ then } \Delta \rhd \varphi \vee \Psi \qquad \textbf{(LG2+)}$$

$$\text{If } \Delta, \Psi \vee \sigma \rhd \varphi \text{ and } \Psi, \text{ then } \Delta, \Psi \rhd \varphi \qquad \textbf{(LG2-)}$$

$$\text{If } \Delta \rhd f(a), \text{ then } \Delta \rhd \exists x f(x) \qquad \textbf{(LG3+)}$$

$$\text{If } \Delta, \exists x f(x) \rhd \varphi \text{ and } f(a), \text{ then } \Delta, f(a) \rhd \varphi \qquad \textbf{(LG3-)}$$

Theorem 13. *For $i \in \{1, 2, 3\}$, **LGi+** and **LGi-** follow from **LGi***,* (**G-FACT**)*, and* (**CUT**).

Proof. For (**LG1+**). Let $\Delta \rhd \varphi$ and $\Lambda \rhd \Psi$. By (**SG4**), it follows that $T\Delta \rhd T\varphi$ and $T\Lambda \rhd T\Psi$. By (**G-FACT**), it follows that $T\varphi$ and $T\Psi$, and by (*Tarski*$^{\rightarrow}$), φ and ψ. If that is so, then, by (**LG1***), $\varphi, \Psi \rhd \varphi \wedge \Psi$ holds. Now, if $\Delta \rhd \varphi$ and $\Lambda \rhd \Psi$, it follows by (**CUT**) that $\Delta, \Lambda \rhd \varphi \wedge \Psi$, as desired.

[14] $T\Delta$ is an abbreviation for $\bigwedge_{\psi \in \Delta} T\psi$.

For (**LG1**-). Suppose that $\Delta, \Psi \wedge \sigma \rhd \varphi$. By (**G-FACT**), $\Psi \wedge \sigma$ holds (which, by the elimination rule for "$\wedge$," can be discomposed in Ψ and σ), and φ also holds. But now, by (**LG1***), it follows that $\Psi, \sigma \rhd \Psi \wedge \sigma$, and so, by (**CUT**), $\Delta, \Psi, \sigma \rhd \varphi$ holds.

For (**LG2**+). Let $\Delta \rhd \varphi$ or $\Delta \rhd \Psi$. Suppose now that $\Delta \rhd \varphi$ holds. By (**G-FACT**), φ obtains. Then, by (**LG2***), it follows that $\varphi \rhd \varphi \vee \Psi$, and by (**CUT**), $\Delta \rhd \varphi \vee \Psi$ also holds. Now suppose that $\Delta \rhd \Psi$ obtains. By (**G-FACT**), Ψ holds. As before, by (**LG2***), $\Psi \rhd \varphi \vee \Psi$, and then, by (**CUT**), $\Delta \rhd \varphi \vee \Psi$ follows. So we conclude that, either by supposing that $\Delta \rhd \varphi$ holds or $\Delta \rhd \Psi$ holds, $\Delta \rhd \varphi \vee \Psi$ obtains as well.

For (**LG2**-). Suppose that $\Delta, \Psi \vee \sigma \rhd \varphi$ and Ψ. By (**LG2***), it follows that $\Psi \rhd \Psi \vee \sigma$. By (**CUT**), $\Delta, \Psi \rhd \varphi$ holds.

For (**LG3**+). Let $\Delta \rhd f(a)$. By (**G-FACT**), $f(a)$ follows. So, by (**LG3***), $f(a) \rhd \exists x f(x)$ obtains. Then, by (**CUT**), $\Delta \rhd \exists x f(x)$ holds as well.

For (**LG3**-). Suppose that $\Delta, \exists x f(x) \rhd \varphi$ and that $f(a)$. Then, by (**LG3***), $f(a) \rhd \exists x f(x)$ obtains. And so, by (**CUT**), it follows that $\Delta, f(a) \rhd \varphi$. ■

Theorem 14. *For $i \in \{1, 2, 3\}$, **WGi** follows from **SGi** and **LGi**-.*

Proof. For (**WG1**). Suppose $T\varphi$ and $T\Psi$. Then, by (**SG1**), $T\varphi \wedge T\Psi \rhd T(\varphi \wedge \Psi)$ holds. But by (**LG1**-), $T\varphi, T\Psi \rhd T(\varphi \wedge \Psi)$ obtains as well.

For (**WG2**). Let $T\varphi$. By (**SG2**), it follows that $T\varphi \vee T\Psi \rhd T(\varphi \vee \psi)$. But now, by (**LG2**-) and from $T\varphi$, it holds that $T\varphi \rhd T(\varphi \vee \Psi)$, as desired.

For (**WG3**). Suppose that $Tf(a)$. By (**SG3**), it follows that $\exists x Tf(x) \rhd T\exists x f(x)$. Finally, by (**LG3**-), $Tf(a) \rhd T\exists x f(x)$ holds as well. ■

Through this last theorem, given the neutral principles, we can derive (**LT1**)—(**LT3***) in the same way as before.

Theorem 15. *For $i \in \{1, 2, 3\}$, **LTi** follows from **SGi**, **LGi**+,* (**G-FACT**)*, and* (**CUT**)*.*

Proof. For (**LT1**). Suppose that $X \models \varphi$ and that $Y \models \Psi$. By (**Translation**), it follows that $E!X \rhd T\varphi$ and $E!Y \rhd T\Psi$. From (**G-FACT**), $T\varphi$ and $T\Psi$ obtain, and by (**SG1**), $T\varphi \wedge T\Psi \rhd T(\varphi \wedge \Psi)$ also holds. But now, from $E!X \rhd T\varphi$ and $E!Y \rhd T\Psi$, it holds by (**LG1**+) that $E!X, E!Y \rhd T\varphi \wedge T\Psi$, and by (**CUT**), $E!X, E!Y \rhd T(\varphi \wedge \Psi)$ obtains as well. Finally, by (**Translation**) again, it holds that $X, Y \models \varphi \wedge \Psi$.

For (**LT2***). Suppose that $X \models \varphi$ or $X \models \Psi$. By (**Translation**), it follows that $E!X \rhd T\varphi$ or $E!X \rhd T\Psi$. By (**G-FACT**), it holds that $T\varphi$ or $T\Psi$, and so, by (**SG2**), that $T\varphi \vee T\Psi \rhd T(\varphi \vee \Psi)$. Now, by (**LG2**+), and from the fact that $E!X \rhd T\varphi$ or $E!X \rhd T\Psi$, it obtains that $E!X \rhd T\varphi \vee T\Psi$. By (**CUT**), $E!X \rhd T(\varphi \vee \Psi)$ holds as well, so by (**Translation**) again, it follows that $X \models \varphi \vee \Psi$, as desired.

For (**LT3***). Let $X \models f(a)$. By (**Translation**), it follows that $E!X \rhd Tf(a)$. By (**G-FACT**), $Tf(a)$ obtains, and so, by (**SG3**), $\exists x Tf(x) \rhd T\exists x f(x)$ holds as well. Now, by (**LG3**+), it obtains from $E!X \rhd Tf(a)$ that $E!X \rhd \exists x Tf(x)$. Applying (**CUT**) to $\exists x Tf(x) \rhd T\exists x f(x)$ and to $E!X \rhd \exists x Tf(x)$, it holds that $E!X \rhd T\exists x f(x)$, and finally, by (**Translation**) again, $X \models \exists x f(x)$ holds as well. ■

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