Theoria, 2025, 40(3), 321-334
https://doi.org/10.1387/theoria.27530

THEORIA
ISSN 0495-4548 – eISSN 2171-679X

What are metainferences?

(¿Qué son las metainferencias?)

Miguel Álvarez Lisboa*

https://orcid.org/0000-0003-0291-4650

Keywords
Substructural logics
Paradoxes
Applied logics
Philosophy of logic

ABSTRACT: In this article I propose a philosophical interpretation of metainferences, that equates them with the reasoning patterns of logicians. This is intended to solve a problem that relates metainferences as objets with metainferences as properties: if both coincide, there seems to be no interest for developping a theory of the former; but if they do not, then this theory seems useless. I claim that my proposal solves the problem, but it also undermines some of the pretended interests of metainferential logics.

Palabras clave
Lógicas subestructurales
Paradojas
Lógicas aplicadas
Filosofía de la lógica

RESUMEN: En este artículo adelanto una interpretación filosófica de las metainferencias, que las identifica con los patrones de razonamiento de quienes se dedican a la Lógica. Esto pretende solucionar un problema que relaciona las metanferencias qua objetos con las metainferencias qua propiedades: si ellas coinciden, entonces no parece haber interés en desarrollar una teoría de las primeras; pero, si no coinciden, tal teoría parece ser espuria. Sostengo que mi propuesta soluciona el problema, pero también debilita el interés que podría tener el desarrollo de lógicas metainferenciales.

1. Introduction

What are metainferences? There are two possible answers in the literature: either they are properties of inferences, or they are new inferential objects on their own right. What is the relation between these two alternatives? This is the question that motivates this article.

I frame the problem as a question about the way one uses a logic. If someone is about to perform according to a metainferential logic, should she abide by the properties of its inferential validities, or by the figures recognized as valid metainferences? Of course, if both coincide, there is no problem, but then a metainferential logics seems rather spurious. It is when they are different that the problem arises.

* ***Correspondence to:*** Miguel Álvarez Lisboa – miguel.alvarez@um.uchile.cl – https://orcid.org/0000-0003-0291-4650

How to cite: Álvarez Lisboa, Miguel (2025). «What are metainferences?»; *Theoria. An International Journal for Theory, History and Foundations of Science*, 40(3), 321-334. (https://doi.org/10.1387/theoria.27530).

Received: 08 May, 2025; Final version: 07 November, 2025.

ISSN 0495-4548 - eISSN 2171-679X /

I will be able to give an answer to the problem, but my solution may seem not very appealing to the metainferentialist. In the end, I argue that the discussion is vitiated by a confusion between metainferences and metalogic, and when these two are taken apart, metainferences appear as much less than what would please to their defenders.

The article is structured as follows. In section 2, I set the stage for the play. Then, in section 3 I push forward the challenge for the upholders of metainferential liberality. In section 4 I propose an answer to this challenge. Section 5 then includes a commentary on what is left out by this solution. A final section summarizes the conclusions.

2. *A stairway to heaven*

A hundred years ago, logic was the study of *logical propositions*: tautologies, or at least theorems derivable in formal systems. Then, around the decade of 1930, some striking results and new developments paved the way for a change of paradigm. Tarski (and, somewhat lately recognized, also Gentzen) made us realize that the key notion in logic is the *consequence relation*.

The (Tarskian) Consequence Relation (T) Let $\mathcal{L}$ be a language. A *consequence relation* is a relation $\vDash\subseteq \wp(\mathcal{L}) \times \mathcal{L}$ that is:

1. Monotonic
2. Reflexive
3. Transitive

During the second half of the XX century the standard became to define a logic as a certain consequence relation of this kind. Call this the Received View on Logical Consequence (RVLC). Later on, certain divergences and liberalizations were introduced. For a start, the single-conclusion framework was extended to a multiple-conclusion setting with disjunctive reading of the conclusions. Also, two kinds of "substructurality" were allowed: the Gentzen-like, that differentiates between inferences with the same premises and conclusions, but in different order or number of repetitions; and the Tarskian-like, that drops one of the three conditions enumerated above (Barrio & Égré, 2022).[1]

Around ten years ago a new trend in the literature around substructurality emerged. The main character of this new discussion was a logic now called ST (Cobreros *et al.*, 2012; Cobreros *et al.*, 2014; Ripley, 2012).[2] The details of its definition are unimportant for the development of my argument, so we may skip the details. It is enough to say that it has the following property:

Fact 1. *An inference* $\Gamma \vdash \Delta$ *is valid in classical logic (*CL*) if and only if is valid in* ST.

And yet it is different from classical logic, as it is non-transitive: under certain definitions of validity you will have that $\Gamma \vdash A$ and $\Gamma, A \vdash B$ are valid in ST , but $\Gamma \vdash B$ is invalid.[3]

---

[1] Monotonicity being the property that links both kinds of substructurality, as it appears at the same time in the seminal papers of Tarski and also corresponds to one of the original Gentzen structural rules.

[2] From an historical point of view, *ST* was discovered several times, under several different forms. I am not interested in the reconstruction of its genealogy. For the time being, it is enough to say that its re-discovery tackled the trend I am going to present and disscuss in what follows.

[3] Throughout this article I shall keep the single-conclusion presentation of inferences and metainferences, but nothing really hinges on this fact.

This property of ST drove the attention to certain expressions that look like inferences, but their relata are already inferences. These were called *metainferences* (inferences between inferences).

Then some logicians from Argentina called for a generalization of this idea. A key passage from their seminal paper reads:

> Yet again, if this is plausible, why stop there? We can definitely consider conse- quence relations between [meta-inferences], and so on and so forth. It can be easily seen how this procedure can be further reproduced, giving us a whole hierarchy of inferences concerned with the logical relations between objects of the lower level(s). (Barrio *et al.*, 2018, p. 106) (with a slight change of terminology).

The idea was further developed in subsequent articles (Barrio *et al.*, 2019; F. M. Pailos, 2020), giving rise to what may be called the Multilevel View of Logical Consequence (MVLC). It is based on the following liberalization of (T):

Hierarchic notion of consequence relation (H) Let $\mathcal{L}$ be a language.

1. An *inference* (metainference of level 0) is a relation $\vdash^0 \subseteq \wp(\mathcal{L}) \times \mathcal{L}$
2. A *meta*^{n}*meta-inference* (metainference of level n+1) is a relation $\vdash^{n+1} \subseteq \wp(\vdash^n) \quad \vdash^n$

A *metainferential logic* is a set of metainferences of all levels.

In 2023 appeared the first textbook on metainferential logics (F. Pailos & Da Ré, 2023). I consider this book as the primary source on the topic, as it resumes and systematizes the literature up to that year. All the details that I will omit can be found or reconstructed from there.

The metainferential project has its supporters and detractors. The firsts take the "ST phenomenon" (Barrio *et al.*, 2018) as but one of the several liberalizations of the relationship between inferences and metainferences that may be exploited for philosophical purposes. The article Barrio *et al.* (2024) is a good illustration of this attitude. On the other side, their detractors see these liberalizations with suspicion. Good examples of cautious attitudes towards metainferential logics may be found in Scambler (2020), Ripley (2021) and Golan (2022).

In the next section I will present a challenge for the MVLC. The argument is not entirely original: it is based on earlier criticisms to the proposal, and traits of it may be found in several places (I give a short note on my main antecedents in section 6). I do not claim complete originality over it. My version is just the one that I shall try to answer in the subsequent sections.

3. *(Not so) user-friendly*

A metainference is an inference between inferences. One may write them down as:

$$(\Gamma \vdash \Delta),\ldots,(\Psi \vdash \Sigma) \vdash (\Xi \vdash \Theta) \tag{1}$$

Or, sometimes better, exploiting the "phenomenology" of Sequent Calculus (as Dicher (2023) would call it):

$$\frac{\Gamma \vdash \Delta \qquad \ldots \qquad \Psi \vdash \Sigma}{\Xi \vdash \Theta}$$

As explained in F. Pailos and Da Ré (2023), chapter 2, there are two ways to interpret these expressions. Metainferences may be:

1. *Properties* of inferences. In this case, what holds true about the inferences determines what metainferences hold.
2. *Objects* on their own right. In this case, different inferential standards may be assorted together to produce very different logics in the spirit of (H).

Of course, the two readings may coincide. Here is one way to assemble a metainferential logic: say the metainferential object α belongs to the metainferential logic, if and only if certain proposition about the validity of its relata holds.

Sympathizers of this approach will usually read expressions like (1) simply as:

$$\text{If } \Gamma \vdash \Delta \text{ is valid, and } \dots \text{, and } \Psi \vdash \Sigma \text{ is valid, then } \Xi \vdash \Theta \text{ is valid}$$

The "and" and the "if... then..." are to be taken as classical connectives and the predicate "valid" has to be assumed to have some explicit definition.[4] Call this the *uniform reading* of metainferences.

The upholders of this approach may go on to say that the development of a theory of metainferences is ill-motivated. In fact, just RVLC and classical mathematics are enough for the development of everything interesting about metainferences. To write down certain metainferences as sequent rules may have some didactic or visual advantages, but offers no further insight and adds nothing substantive to the theory.

This is why one may expect that supporters of MVLC align with the objectual interpretation of metainferences, and moreover defend that these objects should not, in general, follow the uniform reading. We may call this second claim.

Metafreedom: To assembly a metainferential logic you *just* need metainferences (as objects) of every level.[5]

Metainferential logics such as the ones presented in Barrio *et al.* (2019) and Barrio *et al.* (2024), or the ones studied in F. Pailos and Da Ré (2023) all presuppose an implicit acceptance of metafreedom.

We have reached the edge of the problem I want to discuss. To dive right into it, consider the following situation. Suppose $\mathcal{L}1$ is a metainferential logic (in the sense of (H)) that has all the necessary properties to prove the following statement:

Theorem A *If* $\Gamma \vdash^0 A$ *is valid and* $\Gamma \vdash^0 A \rightarrow B$ *is valid, then* $\Gamma \vdash^0 B$ *is valid.*

And yet, that this metainference is not part of $\mathcal{L}1$:

$$\frac{\Gamma \vdash^0 A \qquad \Gamma \vdash^0 A \rightarrow B}{\Gamma \vdash^0 B} \tag{2}$$

Now assume that we are on a situation where we have $\Gamma \vdash^0 p$ and $\Gamma \vdash^0 p \rightarrow q$. The following question rises: is it legitimate to infer q from Γ?

[4] Validity for metainferences is an ice cream that comes in four flavors: local, global, global2 and absolutely global. See F. Pailos and Da Ré (2023) for definitions. None of this is relevant for the current discussion.

[5] The word "metafreedom" appears in print for the first time in Barrio *et al.* (2024). It was coined, to the best of my knowledge, by Camillo Fiore, one of the authors of that article.

1. If the answer is *yes*, then the fact that $\mathcal{L}_1$ does not validate (2) is unsubstantial.
2. If the answer is *no*, then Theorem A is false.

Assuming one does not want to be inconsistent at the metalevel, the only possible answer is the first one. But then the metainferentialist faces the following predicament:

1. If they present a uniform metainferential hierarchy, then the metainferences are unnecessary (metainferences as objects are the same as metainferences as properties; the former are disposable)
2. If they present a metafree hierarchy, then the metainferences are useless (only metainferences (as properties) matter; metainferences (as objects) are unsubstantial).

This is, in essence, the challenge I propose. In more succinct terms: If metainferences (as objects) coincide with metainferences (as properties), then a theory of them is not needed; usual metalogic suffices. If metainferences (as objects) do not coincide with metainferences (as properties), then they cannot be *used*.

The word "use" may have more than one possible explication, but in this context I consider only those that comply with the following desideratum:

Use-as-detachment (UAD) To *use* a (meta-)inferential expression amounts to perform with the conclusion on the basis that certain conditions are met concerning the premises.

Given a valid inference $\Gamma \vdash^0 A$, a non-exhaustive list of possible "uses" considered here may be: to reason from assumptions Γ to a conclusion A; to produce a discourse that upholds A on the basis of Γ; to declare oneself to be able to defend A if being conceded with Γ; to sanction as incoherent to accept Γ and reject A;[6] to behave as if A was true, on the previously acquired belief that Γ are true, etc. Moreover, I assume that UAD and validity are related in the following way:

The Pragmatic Presupposition (PP) The legitimate uses of a logical object depend on it being valid.

The idea is that, no matter what we mean by "use", if $\Gamma \vdash^0 A$ can be used in a certain sense, it is *because* it is valid, according to the logic under consideration. So, to restate the previous problem in more general terms, assume we have a (possibly infinite) set of propositions Γ and certain propositions $A_1, \ldots, A_n$, B. And we have a logic in which:

1. There is a metatheorem about the validity of the *inferences* $(\Gamma \vdash^0 A_1), \ldots, (\Gamma$ f-$^0 A_n)$ and $(\Gamma \vdash^0 \mathrm{B})$.
2. The *meta-inference*

$$\frac{\Gamma \vdash^0 A_1 \quad \ldots \quad \Gamma \vdash^0 A_n}{\Gamma \vdash^0 B}$$

is not valid.

6 Though some have argued that this "bilateralist" reading of inferences is alternative, not subordinate, to the "operational" reading induced by the UAD (Ferguson & Ramírez-Cámara, 2021). All the same, one may still extract from the bilateralist reading of inferences ("It is out of bounds to accept all Γ and reject A") a form of detachment: *If (I accept that) it is out of bounds to accept all* Γ *and reject A, and I accept all* Γ*, then I have to accept A (to remain in bounds).* It is in this sense that I include bilateralism among the admissible explications of "use" that I am concerned with. I am most grateful with an anonymous referee for pointing out the need to stress this point.

3. There is a sense of "use" in which there is a mismatch between the metatheorem being provable and the meta-inference not being valid.

In a scenario like this, in order to answer this question: *should we perform according to the metatheorem, or according to the meta-inference?* The metainferentialist would want to recommend the use of the available metainferences only. But the problem is that metalogic has a preeminence on its own. And the reason is that mathematics is (for most people) true scientific knowledge, and to act according to true scientific knowledge is (I would say, in general) considered a reasonable thing to do. So, its use is legitimate with independence of the metainferential levels of the logic.

A more delicate problem is posed by the reversed situation:

1. We lack a metatheorem about the validity of the *inferences* $(\Gamma \vdash^0 A_1), \ldots, (\Gamma \vdash^0 A_n)$ and $(\Gamma \vdash^0 B)$ (and neither we have a proof of its negation).[7]
2. The *meta-inference*

$$\frac{\Gamma \vdash^0 A_1 \quad \ldots \quad \Gamma \vdash^0 A_n}{\Gamma \vdash^0 B}$$

is valid.
3. There is a sense of "use" in which there is a mismatch between the meta-proposition being not proved and the meta-inference being valid.[8]

In this case the validity of the metainference enables us to perform in a certain way, but the mathematics about the validity of inferences does not authorizes us to do so. Here again, the metainferentialist would want us to reason according to the metainference, but the reasonable attitude is to prevent oneself to do so until a substantial, mathematical proof of the metatheorem is established.

A sensitive example of this second case can be envisaged. In the previous section I mentioned *ST*, the non-transitive version of *CL*. A close relative is *TS*. And we know that:

1. It *is* the case that, if $\Gamma \vdash^0 A$ and $\Gamma, A \vdash^0 B$ are *TS*-valid, then $\Gamma \vdash^0 B$ is *ST*-valid.
2. It *is not* the case that, if $\Gamma \vdash^0 A$ and $\Gamma, A \vdash^0 B$ are *ST*-valid, then $\Gamma \vdash^0 B$ is *ST*-valid.

These are two mathematical, indisputable truths.[9]

We also have two metainferential logics: ST^{ω} and $\widehat{ST}$. These logics share the same set of inferential validities: *S*T. But, as for the first metainferential level, only the first validates Cut:

[7] This second condition is not necessary, but sets the problem under weaker conditions. Of course, if the negation of the metatheorem is true, then the situation is even worse.

[8] I speak loosely about "a metatheorem about the validity", because the precise formulation of this metatheorem depends on the definition of metainferential validity we prefer. As an illustration, if we prefer the "global" approach, then the corresponding metatheorem would be this: "If $(\Gamma \vdash^0 A_1), \ldots, (\Gamma \vdash^0 A_n)$ are valid, then $(\Gamma \vdash^0 B)$ is valid". Whereas if we prefer the "local" approach, the corresponding metatheorem is: "There is no model of $(\Gamma \vdash^0 A_1), \ldots, (\Gamma \vdash^0 A_n)$ that is a countermodel of $(\Gamma \vdash^0 B)$". (Again, see F. Pailos and Da Ré (2023) for details.) The dilemma only needs "a mismatch" between the metatheory and the accepted metainferences; so, for instance, if metainferences are accepted *qua* globally valid, then it is the global reading of the metatheorem the one that should not hold.

[9] Modulo a suitable definition of validity. See the previous footnote.

$$\frac{\Gamma \vdash^0 A \qquad \Gamma, A \vdash^0 B}{\Gamma \vdash^0 B}$$

The reason is that, in ST^{ω}, the valid metainferences are those that preserve TS-satisfaction to ST-satisfaction, thereby accommodating the first of the facts stated above. Whereas in $\widehat{ST}$ metainferences are those that preserve ST-satisfaction.[10]

Now, imagine a situation where we are told to perform according to ST^{ω} -validity[11] and we are given that $\Gamma \vdash^0 p$ and $\Gamma, p \vdash^0 q$. Is it legitimate to conclude that $\Gamma \vdash^0 q$?

The metainferentialist would say: "Yes. In your logic, Cut is a valid metainference. Therefore, you can use it and conclude that $\Gamma \vdash^0 q$." But this might raise some eyebrows.

One possible line of complaint could be this: "it is true that my logic validates Cut, but this is because, if $\Gamma \vdash^0 p$ and $\Gamma, p \vdash^0 q$ are TS-valid, then $\Gamma \vdash^0 q$ is ST-valid. But, in order to perform this Cut, I would need to have instances of the premises, and I do not have them. My logic is ST^{ω}; therefore, I do not accept $\Gamma \vdash^0 p$ and $\Gamma, p \vdash^0 q$ as TS-validities, but rather as ST-validities."

The metainferentialist may reply: "There is no such thing as having ST's or TS-validities. Logics only have inferences and metainferences. Their standards (ST, TS, CL, . . .) are not part of the hierarchy. They are only relevant in the process of defining the extension of the logic, but they do not belong to the inferences themselves. So, you do have $\Gamma \vdash^0 p$ and $\Gamma, p \vdash^0 q$ (because they are ST-validities), and you also have $\Gamma \vdash^0 q$ (because you have the premises of a rule that allows you to conclude it)."

But then we are thus left with the following situation: we know that $\Gamma \vdash^0 p$ and $\Gamma, p \vdash^0 q$. Then, the metainferentialist tells us that we can conclude $\Gamma \vdash^0 q$, but our mathematical knowledge about ST does not back up this recommendation.

We will take a closer look at an actual example of this kind in section 5.

4. *What metainferences are*

In the previous section I put forward a challenge for the upholders of the MVLC. In this section I propose a solution. Still, it may be much *less* than what a true metainferentialist would want, but about this I will say a few things in the next section.

The situation so far is this: the metainferentialist claims that a logic has not only valid inferences, but also valid metainferences. This needs to amount to something other than what we already have (inferences and classical mathematics), if this position is going to be an interesting new paradigm in philosophical logic. So she needs to sustain that metainferences are not just a fancy way of presenting usual metatheorems about inferences. But, the sooner she introduces some brand new systems, they seem nonetheless useless, as when the time comes to reason with them, the most reasonable way to perform is still to follow pure inferential logic plus its classical metatheory.

I find this line of critique essentially correct: inferential logics are theories of valid reasoning, and its classical metatheory is true scientific knowledge. The wisest advice is to always perform according to true scientific knowledge; therefore, when

[10] See F. Pailos and Da Ré (2023) for a detailed exposition of this machinery.

[11] This may sound a little odd, as usually we do not "choose" a logic to work with. But it makes perfect sense, for instance, if the reasoner is not a person but a machine.

using a logic, we should look always and only to the metatheory of that logic. But metainferences are *not* what determines how to use inferences. Cut, as a rule in a sequent calculus, is *not* a metainference. Transitivity, as a property of a consequence relation, is *not* a metainference either. (It *is* a metainference (as property), but not a metainference (as object), despite its misleading shape). Metainferences are *part* of the theory of valid reasoning and not a *metatheory* of it.

Let's step back a little to see the picture in full. Inferences are figures that pretend to be generalizations of reasoning patterns. Modus Ponens:

$$A, A \rightarrow B \vdash B$$

is supposed to be the general form of certain arguments, discourses or thought processes. I claim that metainferences are also generalizations of reasoning patterns, only that in this case the propositions involved are generalizations of other reasoning patterns. They *add* schemes to the inferential ones, and are not *about* them.

Yet Modus Ponens belongs to a type of pattern (from propositions to propositions), that one usually encounters in rational discourse. One may say: there seems to be no instances of arguments from inferential schemes to inferential schemes in the vernacular practice. But here's the catch, for my claim is that there are. They are the typical kind of argument held by *logicians*.

Here's a real(istic) example.[12] In the sixties, Newton da Costa wanted to create a logic that tolerates contradictions. To this end, he envisaged a logic that invalidates the law of non-contradiction (LNC), and what he came up with was a weakening of classical logic that lacks Explosion. Later on, at the end of the eighties, Graham Priest and Richard Routley criticized his approach by saying only the failure of Explosion was necessary, not the failure of LNC.

We may formalize the discussion as this. Da Costa claims that:

$$\neg A \rightarrow \neg B$$

And Priest and Routley complain that:

$$\neg(\neg A \rightarrow \neg B)$$

Where A is "Explosion is valid" and B is "LNC si valid". But this formalization looses some very relevant information: that we are dealing not with plain propositions, but with propositions about validity. So, a more accurate formalization of the former would be:

$$\frac{(A, \neg A \nvdash^0 B)}{\left(\nvdash^0 \neg(C \wedge \neg C)\right)}$$

And the formalization of the latter would be the negation of this:

$$(A, \neg A \nvdash^0 B) \nvdash^1 \left(\nvdash^0 \neg(C \wedge \neg C)\right)$$

[12] I am taking the reconstruction of the discussion as exposed in Slater (1995). All I need is that it serves as a plausible example of an actual debate in contemporary Logic, not that it is completely fair and accurate.

Note that neither of these is a plain mathematical truth. In other words, they do not constitute scientific knowledge per se; they are both schemes, patterns of reasoning about certain mathematical objects (to wit, inferences). In fact, you may have paraconsistent logics where the first represents a true metatheorem (da Costa's C^{ω}, for instance) and logics where the second represents a true metatheorem (Priest's LP).

And what about metainferences of higher levels? Is it realistic to expect that some day logicians would enter a dispute over the 56th metainferential level? Surely not. But this is not a problem. Once one accepts the level of abstraction that formalizations deliver, one accepts to account for cases that are very unlikely to ever instantiate in real life. The situation is the same as when Wittgenstein asks himself, in the *Tractatus*: "How could we decide a priori whether, for example, I can get into a situation in which I need to symbolize with a sign of a 27-termed relation?" (TLP 5.5541). The conclusion there (and here) is that the formalism is not bounded by its applications, but the other way around. "Empirical reality is limited by the totality of objects. The boundary appears again in the totality of elementary propositions. The hierarchies are and must be independent of reality" (TLP 5.5561). Wittgenstein is certainly not talking about *metainferential* hierarchies, but the general idea applies to our case.

All this is to say that metainferences, just as inferences, correspond to reasoning patterns. But this is still not evidence in favor of metafreedom. In fact, one may say that even the paraconsistent logicians mentioned above are thinking on the classical closure of their inferential theories, thus abiding to the reading of metainferences as properties. If this is the case, then even if metainferences are legitimate logical objects, they should not diverge from the classical meta-closure of the inferential level.

According to the interpretation I propose here, a metainferential logic is a theory of reasoning patterns that include the reasoning patterns about simpler reasoning patterns. This is typically what logicians do: they reason and argue about reasoning patterns of people. Metafreedom is the acknowledgement that these patters may not coincide with the simpler ones.

Why would someone make such a deviance? Well, simply because logicians usually take advantage of these liberties to solve convoluted problems. Take, for instance, the proposition PREM, used by Gillian Russell to account for the failure of Reflexivity (Russell, 2018):

PREM This proposition is the premise of this argument.

The inference

$$\text{PREM} \vdash^{0} \text{PREM}$$

Has a true premise and a false conclusion, so it is a countermodel to the scheme $A \vdash^0 A$.

A generalization of the above proposition gives us:

PREM (x) This proposition is

$$\underbrace{\text{the conclusion of...}}_{x \text{ times}}$$

the premise of this argument.

So PREM(0) = PREM, but PREM(1) ("This proposition is the conclusion of the premise of this argument") habilitates a metainferential paradox, as it accounts for the failure of metareflexivity:

https://doi.org/10.1387/theoria.27530

$$\frac{\vdash^0 \textsc{prem}(1)}{\vdash^0 \textsc{prem}(1)}$$

And PREM(2) gives a meta-metainferential paradox... and so on. Nobody but logicians would ever care about this kind of propositions; but logicians, indeed, care a lot. And this is only one of the many combinations and iterations of premise-conclusion containment we can design. Therefore, there is a plethora of different metainferential paradoxes to account for, and plenty of space to propose different solutions to them. Just as *ST* is a clever solution to the Liar paradox (another proposition that, allegedly, only logicians care about), the metainferential logics may be nice ways to solve these multilevel paradoxes. And solving paradoxes is one prominent example of what logicians do.

Incidentally, note that a construction containing the phrase "...the premise of the conclusion of the premise of...", or something like this, can only refer to an object that is a nested inferential expression. Therefore, a metainference.

5. *What metainferences are not*

In the previous section I have claimed that logical theories are accounts of inferential patterns, and that metainferential logics are accounts of inferential patterns that include the kind of inferences that logicians use. And I have proposed that this solves the challenge proposed in section 3, because at each point of a metainferential hierarchy there's no question as to how a logic should be used: in each case, you should only consider what you can *prove* about the elements in that level. For only that is true scientific knowledge.

Yet, one may ask if this is really a satisfying answer for the upholders of the MVLC. My guess is that probably not. And I have a good example to illustrate this point.

In Barrio *et al.* (2024), various metafree logical theories are presented. And in the last section, the authors claim that these logical systems may be attractive to those logicians that are prone to the *hypocrisy objection*. In a nutshell, the latter is this: if you defend that a certain logic is correct, then you should abide by it as a standard for proving things about it. If you fail to do so, you are being hypocrite about the validity of your logic. Here is an example. Say you defend a logic $\mathcal{L}1$. This logic lacks the reversed contraposition principle:

$$(\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)$$

But then you want to prove completeness, and you go for the popular Henkin strategy. This strategy, as is well known, consist in showing that a countermodel can be extracted from a failed attempt to construct a proof. But, to say that this amounts to proving completeness, is an instance of the reversed contraposition principle. So you are using a logical strategy that your logic deems as invalid.

Among the many alternatives that non-classical logicians have to reply to this objection is the *recapture strategy*. The idea is to show that, although some generalized forms of classical principles may fail in their logic, some instances of them (crucially, those they need) are nonetheless still sound. Here's where the authors kick in:

> Many of our meta-classical non-classical logics can be viewed as providing a novel and elegant kind of recapture result. Consider those of our systems that recover positive aspects of classical logic (viz. validities) as opposed to negative aspects (antivalidities). These are all the **mc**-logics, the **eq**-logics and **uLP**. All these systems allow the non-classical logician to stick with her preferred non-classical notion of validity for inferences, while at the same time recovering, by means of the appropriate metainferences, any piece of classical reasoning she wants to perform. (Barrio *et al.*, 2024, p. 20)

We do not need to go that deep into the definitions to see what is going on here. It is enough to know that, in the logic **mcLP**:

1. $A, A \to B \nvdash^0 B$, because Modus Ponens fails in *LP*.
2. $\varnothing \vdash^1 (A, A \to B \vdash^0 B)$, because Modus Ponens holds in *ST*.

Now the authors say the following. I will quote them in length, so I make sure to do not misrepresent their thinking in the reconstruction.

> Suppose that, in the process, she needs to make an inferential transition from p and "if *p* then *q*" to *q*. *A classical* logician would justify this transition by invoking the fact that the inference $p, p \to q \vdash^0 q$ is valid in CL and the premises *p* and $p \to q$ hold. Now, since our logician uses **mcLP**, her justification is a bit different: she invokes the fact that the metainference
>
> $$\frac{\varnothing}{p, p \to q \vdash^0 q}$$
>
> is valid in **mcLP**, all the premises of the metainference hold, and so do *p* and $p \to q$. Even though the justification is different, the sentence being inferred is in both cases q (and not some meta-variant of it). So, the **mcLP** logician has reached the same conclusion as her classical fellow. From there on, the **mcLP** logician can keep mimicking the classical logician in this way, obtaining the same conclusions as him at each step. At the end of the reasoning, both logicians will have arrived at the claim they were aiming at. (Barrio *et al.*, 2024, p. 21) (With my notation.)

Here the authors go straight into the aporia presented at the end of section 3. If the authors are right and "the **mcLP** logician —call her Sophia— can keep mimicking the classical logician in this way, obtaining the same conclusions as him at each step", then she has to conclude that from λ and $\lambda \to \bot$ follows $\bot$, as the classical logicians do.[13] But she endorses the inferential validity of *LP*, and people usually like *LP* because it is non-explosive in face of paradoxes. In fact, she should *not* be able to perform[14] the inference, because the conclusion in

$$\frac{\varnothing}{\lambda, \lambda \to \bot \vdash^0 \bot}$$

Is an *ST*-validity, not an *LP*-validity, and she is not committed to follow *ST*, but *LP*, in her inferential practices. So we have that she both should, and should not, accept $\bot$ on the basis of λ and $\lambda \to \bot$.

The previous analysis is vitiated by the presumed belief that metainferences justify uses (performances). This is not the case. If Sophia performs in accordance to **mcLP**, then from λ and $\lambda \to \bot$ she does not conclude $\bot$. Also, for her, logicians conclude $\lambda, \lambda \to \bot \vdash^0 \bot$ from $\varnothing$ ($\bot \vdash^0 \bot$). She is a logician herself, so she does conclude $\lambda, \lambda \to \bot \vdash^0 \bot$ from $\varnothing$; but this is not a new rule that she can apply again. It is, so to speak, a terminal piece of knowledge. She is not forced to perform according to her own metainferential theory, because this is not what the metainferential theory is about in the first place.

Some may want to read from this that her practices are not "closed under her own rules" (Scambler, 2020), and so she is, indeed, a "hypocrite" in the sense above sketched. But I do not think that this is the case either. She is being perfectly consistent with her accepted theory: she do believes that $\lambda, \lambda \to \bot \vdash^0 \bot$ is a an *ST*-validity. She does not perform the in-

[13] Here λ is The Liar ("This proposition is false"), or a similar paradox-inducing sentence. λ is explosive in classical logic, but not in *LP*.

[14] Their word, not mine.

ference though, because she is not an *ST*-reasoner, but an *LP*-reasoner. But, in any case, this is not a "novel and elegant" kind of recapture strategy.[15]

My proposal, in sum, amounts to establishing a clear distinction between three layers or dimensions:

1. The *inferential dimension*, where inferential logics in the spirit of RVLC belong. These are theories on how people infer.
2. The *metainferential dimension*, where metainferential logics in the spirit of MVLC belong. These are theories on how people infer, and also on how logicians infer about how people infer.
3. The *metalogic dimension*, where there is pure and simply standard mathematics. It constitutes true scientific knowledge, and you should abide by it in your rational performances.

Logicians that tend to confuse the second and third layer apparently get seduced by the fact that metainferential expressions (specially those presented in Gentzen format) are ways to write down metalogical theorems. But this is the exact same confusion that got Achilles trapped in the endless race with the Tortoise (Carroll, 1895). Just as Modus Ponens is not a proposition, Meta-Modus Ponens is not a rule of inference, Cut is not the property of transitivity, and so on.

6. *Conclusions*

The growing interest on metainferential logics calls for a clarification on what metainferences are. To some authors, they are just properties of inferences; to others, they are inferential objects on their own. I defend that, even when one takes the second alternative, the way a logic is used shall always and only follow the properties of the inferences; but this does not mean that metainferences are just silent effigies to decorate our logical theories. My claim is that they habilitate new logical problems, and consequently also enable original solutions to these problems. They are interesting tools for logicians to keep exploring their typical philosophical problems. And if they ever connect or represent something from the vernacular practice, it is the kind of arguments that logicians themselves use when thinking and speaking about reasoning patterns of other people. They are, quite literally, "inferences about inferences".

My overall take on all this aligns with the one put forward in Tajer (2021). The basic idea is that non-classical logicians most usually do not challenge the scientific status of classical logic. They propose new logical systems to solve problems, but use classical logic because this is the system that represents the reasoning patterns of everyday standard mathematics. In his words:

> These philosophical logicians solve specific problems such as the Liar paradox, explain their non-classical solutions using classical logic in the meta-language (typically with a fixed-point theorem), show why classical logic can be maintained in non-semantic reasoning, and provide additional responses to revenge problems. Therefore, the contemporary logical development does not look like a battle between rival hypotheses, but more like the exploration of an unknown land, where discovering new points of view has an intrinsic value, provided the agents follow some common rules. (Tajer, 2021, p. 133)

We may consider the science-fiction scenario where a society evolved to have a mathematical practice that does not abide by classical logic, but by (say) **mcLP**. What this means, to me, is that mathematicians and rational people in this society reason according to LP in their usual logical situations. While, at the same time, the logicians in the Philosophy

[15] In fact, compare the passage from Barrio *et al.* (2024) that we are commenting with Scambler (2020), section 5.

departments and in the faculties of Mathematics and Computer science reason about logic following the patterns of *TS*/*ST*. At a certain point, some philosopher may notice that something does not add up in her society. Why are philosophers talking about one thing, while people in fact reason according to another thing? And this may be a very interesting topic of research for her, but —and this is the crucial point— this does not deem her reality as logically impossible. This is what metafreedom amounts to: to recognize that it is not inconsistent to accept a theory of inferences, and also a theory of metainferences that diverges from the metatheory of the former.

Time will say if this was an interesting possibility to explore in the first place, or not.

Just one last thing before the close. As I acknowledged at the end of section 2, the challenge I proposed to the MVLC in this article is not entirely original. It is an argument that has been haunting for some time the community working on MVLC, and (depending on the personality and mood of the people involved) has taken different shapes. *My version* of the argument has three main antecedents: Ripley (2021), Golan (2022), and Scambler (2020). In the first it is argued that endorsing an inferential logic does not imply endorsing a metainferential logic. The second fed me with the illuminating observation that non-transitive metainferential logics block the passage from lemmas to new theorems, and so in ST^{ω} there is no room for "real inferential action". My notion of "use" is inspired by this idea. Finally, the third makes a case on to how metainferential logics can be not closed under its own rules, an idea that played a prominent role in section 5.

Acknowledgements

This article is based on one of the two main arguments of my doctoral thesis, enriched with some very useful comments and critiques received during the viva voce. I would like to acknowledge my debt to Dr. Eduardo A. Barrio, who supervised that work, and the examiners, Dr. Federico Pailos, Dr. Ricardo Rodríguez and Dra. Lourdes González del Carmen Huesca for their insightful feedback. I am also grateful to the Buenos Aires Logic Group, the thriving intellectual community with whom I had the pleasure to work during my doctoral studies.

REFERENCES

Barrio, E., & Égré, P. (2022). Editorial introduction: Substructural logics and metainferences. *Journal of Philosophical Logic*, *51*(6), 1215-1231.

Barrio, E., Fiore, C., & Pailos, F. (2024). Meta-classical non-classical logics. *The Review of Symbolic Logic*, *17* (4), 1146-1171.

Barrio, E., Pailos, F., & Szmuc, D. (2018). What is a paraconsistent logic? *Contradictions, from consistency to inconsistency* (pp. 89-108). Springer. https://doi.org/10.1007/978-3-319-98797-25

Barrio, E., Pailos, F., & Szmuc, D. (2019). (Meta)inferential levels of entailment beyond the Tarskian paradigm. *Synthese*, 198 (S22), 5265-5289.

Carroll, L. (1895). What the tortoise said to achilles. *Mind*, *4*(14), 278-280.

Cobreros, P., Egré, P., Ripley, D., & van Rooij, R. (2012). Tolerant, classical, strict. *The Journal of Philosophical Logic*, *41*, 347-385. https://doi.org/10.1007/s10992-010-9165-z

Cobreros, P., Égré, P., Ripley, D., & van Rooij, R. (2013). Reaching Transparent Truth. *Mind*, 122(488), 841-866. https://doi.org/10.1093/mind/fzt110

Dicher, B. (2023). Substructural heresies. *Inquiry. An Interdisciplinary Journal of Philosophy*. https://doi.org/10.1080/0020174X.2023.2254816) [Published online].

Ferguson, T. M., & Ramírez-Cámara, E. (2021). Deep ST. *Journal of Philosophical Logic*, *51*(6), 1261-1293.

Golan, R. (2022). On the metainferential solution to the semantic paradoxes. *Journal of Philosophical Logic*, *52*(3), 797-820.

Pailos, F., & Da Ré, B. (2023). *Metainferential logics*. Springer Cham.

Pailos, F. M. (2020). A fully classical truth theory characterized by substructural means. *The Review of Symbolic Logic*, *13*(2), 249-268. https://doi.org/10.1017/S1755020318000485

Ripley, D. (2012). Conservatively extending classical logic with transparent truth. *Review of Symbolic Logic*, *5*(2), 354-378.

Ripley, D. (2021). One step is enough. *Journal of Philosophical Logic*, 1-27.

Russell, G. (2018). Logical nihilism: Could there be no logic? *Mathematical Issues*, *28*(1), 308-324.

Scambler, C. (2020). Classical logic and the strict tolerant hierarchy. *Journal of Philosophical Logic*, *43*(2), 351-370.

Slater, B. H. (1995). Paraconsistent logics? *Journal of Philosophical Logic*, *24*(4), 451-454. Retrieved May 2, 2023, from http://www.jstor.org/stable/30226556

Tajer, D. (2021). Logic as a puzzle-solving activity. *Análisis Filosófico*, *41*(1), 119-140.

Miguel Álvarez Lisboa has a PhD by the University of Buenos Aires. His main topics of research are Philosophical logics, with a focus on Dialogics, intuitionistic logic and metainferences. He is currently working as a freelance Logic teacher.

Address: www.migueldaclasesdelogica.com — E-mail: miguel.alvarez@um.uchile.cl – ORCID: 0000-0003-0291-4650