A Characterization of Logical Constants *Is* Possible

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ABSTRACT: The paper argues that a philosophically informative and mathematically precise characterization is possible by (i) describing a particular proposal for such a characterization, (ii) showing that certain criticisms of this proposal are incorrect, and (iii) discussing the general issue of what a characterization of logical constants aims at achieving.

Keywords: logical constant, logical consequence

Can there be a mathematically precise and philosophically informative characterization of logical constants? Gómez-Torrente's answer to this question is negative:

> Logicians and philosophers of logic have often tried to offer philosophically richer characterizations of the notion of a logical expression than the one(s) suggested by the vague pragmatic principles just mentioned. These attempts usually characterize the notion of a logical expression ... in terms of alleged semantic, epistemic or mathematical peculiarities of the logical expressions. I conjecture that these attempts will not succeed, since it must be nearly impossible to model closely the vague and pragmatic notion of a logical expression in those terms. [Gómez-Torrente (2003)]
>
> One of the philosophical conclusions that seem to me to receive strong support from the [above criticisms of philosophical theories of logical constants] ... is that most (perhaps all) of the substantive philosophical conceptions of the problem of logical constants may have created unsolvable versions of the problem. The search for a characterization of the intended set of logical expressions ... may be a hopeless project if it is required ... that the characterization be given in terms of mathematical concepts ... or unexplicated semantic and epistemic properties ... [Gómez-Torrente (2002), 31-32]

I agree with Gómez-Torrente's claim that our choice of logical constants involves, and should involve, pragmatic considerations, but I think this does not rule out the possibility, or desirability, of a mathematically precise and philosophically informative characterization of these constants. In this paper I will describe one proposal for a precise characterization of logical constants, explain why Gómez-Torrente's criticism of this proposal is incorrect, and discuss the more general issue of what a characterization of logical constants aims at achieving.

1. A Characterization of Logical Constants

The characterization I will defend is one I proposed in Sher (1991, Ch. 3), incorporating elements from Mostowski (1957), Tarski (1966) and Lindström (1966):

> DEFINITION LT *C* is a ... *logical term [constant]* iff C is a truth-functional connective or *C* satisfies [the following] conditions ... on logical constants [Sher (1991), 56]:
>
> A. A logical constant *C* is syntactically an *n*-place predicate or functor (functional expression) of level 1 or 2, *n* being a positive integer.
> B. A logical constant *C* is defined by a single extensional function and is identified with its extension.

C. A logical constant C is defined over models. In each model **A** over which it is defined, C is assigned a construct of elements of **A** corresponding to its syntactic category. Specifically I require that C be defined by a function f_C such that given a model **A** (with universe A) in its domain:
   a. If C is a first-level n-place predicate, then $f_C(\mathbf{A})$ is a subset of A^n.
   b. If C is a first-level n-place functor, then $f_C(\mathbf{A})$ is a function from A^n into A.
   c. If C is a second-level n-place predicate, then $f_C(\mathbf{A})$ is a subset of $B_1 \times \ldots \times B_n$, where for $1 \leq i \leq n$,

$$B_i = \begin{cases} A & \text{if } i(C) \text{ is an individual} \\ \boldsymbol{P}(A^m) & \text{if } i(C) \text{ is an } m\text{-place predicate} \end{cases}$$

   ($i(C)$ being the ith argument of C).
   d. If C is a second-level n-place functor, then $f_C(\mathbf{A})$ is a function from $B_1 \times \ldots \times B_n$ into B_{n+1}, where for $1 \leq i \leq n+1$, B_i is as defined in (c).

D. A logical constant C is defined over *all* models (for the logic).

E. A logical constant C is defined by a function f_C which is invariant under isomorphic structures. That is, the following conditions hold:
   a. If C is a first-level n-place predicate, **A** and **A'** are models with universes A and A' respectively, $<b_1,...,b_n> \in A^n$, $<b'_1,...,b'_n> \in A'^n$, and the structures $<A,<b_1,...,b_n>>$ and $<A',<b'_1,...,b'_n>>$ are isomorphic, then $<b_1,...,b_n> \in f_C(\mathbf{A})$ iff $<b'_1,...,b'_n> \in f_C(\mathbf{A'})$.
   b. If C is a second-level n-place predicate, **A** and **A'** are models with universes A and A' respectively, $<D_1,...,D_n> \in B_1 \times \ldots \times B_n$, $<D'_1,...,D'_n> \in B'_1 \times \ldots \times B'_n$, (where for $1 \leq i \leq n$, B_i and B'_i are as in (C.c)), and the structures $<A,<D_1,...,D_n>>$, $<A',<D'_1,...,D'_n>>$ are isomorphic, then $<D_1,...,D_n> \in f_C(\mathbf{A})$ iff $<D'_1,...,D'_n> \in f_C(\mathbf{A'})$.
   c. Analogously for functors. [*Ibid.,* 54-5]

2. Gómez-Torrente's Criticism

According to Gómez-Torrente, the above definition provides a necessary but not a sufficient condition for logical constancy. His criticism proceeds in three steps:

1. Tarski's (1966) definition of logical constants (a restricted version of my condition (E)) fails:

> [T]he question whether Tarski's definition provides a *sufficient* condition for logical constancy, even for the classical quantificational languages for which it is intended, seems to have a clear negative answer. A counterexample would be produced if, so to speak "by chance", some constants not typically treated as logical denoted notions invariant under permutations in all universes of discourse. Persuasive counterexamples of this kind result if we consider predicates not treated as logical and which are predicated *falsely* of all individuals in all universes. These predicates denote a logical notion in all universes, namely the empty set (often, they denote the empty set in all possible universes). 'Unicorn', 'heptahedron', and 'male widow' are good candidates; by 'heptahedron' I abbreviate 'regular polyhedron of seven faces'[1]. [Gómez-Torrente (2002), 18]

[1]Henceforth I will assume that this is the English meaning of 'heptahedron'.

2. Sher (1991) – i.e., my characterization – purports to overcome the problem with Tarski's definition by introducing Condition (B):

> Gila Sher [1991] has proposed a definition of logical constancy ..., and she has considered an objection to her proposal essentially analogous to the one based on the existence of terms like 'unicorn', 'heptahedron', etc. In order to deny relevance to this objection, Sher says that under her proposal, a logical term is *identified* with the (class)-function that assigns to every (set)-universe the denotation of the term in the universe:
>
>> logical terms are identified with their (actual) extensions, so that in the metatheory the definitions of logical terms are rigid. (...) Their (actual) extensions determine one and the same function over models, and this function is a legitimate logical operator. (...) ... we may say that the only way to understand the meaning of a term used as a logical constant is to read it rigidly and formally, i.e., to identify it with the mathematical function that semantically defines it ([[Sher (1991),] 64-65]).
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> [Gómez-Torrente (2002), 18-19]

3. Condition (B) fails to overcome the problem with Tarski's definition:

> This counterobjection is highly problematic. Suppose we had a primitive one-place logical predicate '∅' in mathematical languages, whose meaning was given simply by the stipulation that it abbreviates the expression 'is not identical with itself'. Under Sher's proposal, '∅', 'unicorn', 'heptahedron', 'male widow', etc. are all the same term, and hence the sentences '∀x~unicorn(x)', '∀x~heptahedron(x)' and '∀x~male widow(x)' must be as logically true (in the intuitive sense) as '∀x~∅x' (and '∀x~(x≠x)') would appear to be. ... Since sentences like '∀x~unicorn(x)', '∀x~heptahedron(x)', '∀x~male widow (x)' ... are *not* logical truths, in any traditional sense of 'logical truth', it seems clear to me that Sher's move is not acceptable as a way of meeting the obvious objection posed by terms like 'unicorn'. [*Ibid.,* 19]

3. Rebuttal

It is a well-known and obvious fact that truth is relative to the use of words. If 'unicorn' were used as 'elephant' is commonly used in English, the sentence '∀x~unicorn(x)' would be false. It is also quite obvious that the sentences '∀x~unicorn(x)', '∀x~heptahedron(x)' and '∀x~male widow (x)' are not logically true *when their English terms are used as they are ordinarily used in English*: the truth of '∀x~unicorn(x)' is based on a zoological fact, that of '∀x~heptahedron(x)' on a geometrical fact, and that of '∀x~male widow (x)' on a linguistic fact. Are they logically true when their English terms are used in a non-standard way? That depends on how the terms are used. If 'unicorn' is used in the way 'non self identical' is commonly used in English, then (under this use) '∀x~unicorn(x)' *is* logically true. All this is compatible with my proposal. It follows from this proposal that the terms 'unicorn' 'heptahedron' and 'male widow', *as they are commonly used in English*, are extra-logical rather than logical, and as such they are "strongly variable", i.e., every formally possible extension compatible with their syntactic category is assigned to them in some model (Sher 1991: 47-8). In particular, these constants are assigned non-empty extensions in some models; in these models the sentences '∀x~unicorn(x)', '∀x~heptahedron(x)' and '∀x~male widow (x)' are false; hence, these sentences do not come out logically true. Does the proposal *advocate* a non-standard use of English terms? No. The proposal offers a set of conditions for using a term as a logical constant in a given language, but it does not attempt to reform ordinary English or to legislate how a term like 'unicorn' should be used in semi-artificial languages (or how a symbol whose meaning is defined using a term like 'unicorn' is to be used in an artificial language).

Gómez-Torrente too, does not purport to legislate the use of terms in natural or artificial languages. He does not say that it is impossible, or forbidden, to use terms in a non-standard way, e.g., to use 'unicorn' as 'non self identical'. What he says is that an adequate characterization of logical constants for a natural language like English, or for a semi-artificial language in which the natural-language expressions have their usual meaning, (or for a fully-artificial language in which certain symbols are defined using expressions like 'unicorn', 'heptahedron' and 'male widow', used as in ordinary English,) must be done in accordance with the observation that in such languages sentences like '∀x~unicorn(x)', '∀x~heptahedron(x)' and '∀x~male widow (x)' are not logically true. And he is right. But he claims that the theory developed in Sher (1991) renders such sentences logically true. And this claim is wrong. To see where the error lies, let us examine how Gómez-Torrente arrives at this claim. His argument (as formulated in the passage cited above) appears to be:

1. The constants 'unicorn', 'heptahedron' and 'male widow' are logical according to Sher's (1991) characterization of logical constants.
2. As logical constants, 'unicorn', 'heptahedron' and 'male widow' are defined by functions, $f_{unicorn}$, $f_{heptahedron}$, and $f_{male\text{-}widow}$, which assign to every model **A** the empty set as its value.
3. As a result, the sentences '∀x~unicorn(x)', '∀x~heptahedron(x)' and '∀x~male widow(x)' are true in all models.
4. It follows that these sentences are logically true according to Sher's theory.

This argument, however, relies on an erroneous premise, namely (1). Why does Gómez-Torrente believe that (1) is true? His argument for (1) appears to be the following:

a. '∅' is a logical constant according to the definition of logical constants in Sher (1991).

b. Under Sher's proposal '∅', 'unicorn', 'heptahedron' and 'male widow' are all the same term.

c. Therefore, under Sher's proposal, 'unicorn', 'heptahedron', and 'male widow' are logical constants.

But this argument, too, is based on an erroneous premise, namely, (b).

Why does Gómez-Torrente believe that (b) holds according to Sher (1991)? He does not say. My conjecture is that he reasons as follows:

(i) According to Sher (1991) logical constants are identified with their extension.

(ii) The extension of '∅' is the empty set, the extension of 'unicorn' is the empty set, the extension of 'heptahedron' is the empty set, and the extension of 'male widow' is the empty set.

(iii) Therefore, according to Sher (1991), '∅', 'unicorn', 'heptahedron' and 'male widow' are the same term.

But (iii) does not follow from (i) and (ii). What does follow from (i) and (ii) is:

(iii') If 'unicorn', 'heptahedron' and 'male widow' are logical constants according to Sher's definition, and if their extensions in all models are the same as in the actual world, then they are the same logical constant as '∅'.

But (iii') is a different conclusion from (iii).

The correctness of Gómez-Torrente's criticism thus depends on whether 'unicorn', 'heptahedron' and 'male widow', as they are used in the sentences above, are logical constants according to my proposal. I will show that they are not.

1. 'Unicorn'. Let '∀x~unicorn(x)' be a sentence of a language L in which 'unicorn' has its usual English meaning. According to (LT), 'unicorn' is a logical constant of L iff either it is a truth functional connective of L or it satisfies the conditions (A)-(E) on logical constants in L. Clearly 'unicorn', as it is used in ordinary English, is not a truth-functional connective (or any other kind of connective). The question is whether it satisfies the conditions (A)-(E) in L. Let us examine these conditions one by one.

Condition A. Condition (A) says that a logical constant is syntactically an n–place predicate or functor of level 1 or 2. 'Unicorn' is a 1-place predicate of level 1; as such it satisfies condition (A).

Condition B. Condition (B) says that a logical constant is defined by a single extensional function and is identified with its extension. The nature of this extensional function is specified in Condition (C).

Condition C. Condition (C) says that a 1-place logical predicate of level 1, *C,* is defined by a function f_C over models, such that for every model **A** with a universe *A* in its domain, f_C assigns to **A** a subset of *A*. The domain of f_C is specified in Condition (D).

Condition D. Condition (D) says that the domain of f_C is the class of *all* models for the given language; in particular, any set whatsoever, whether of existent or fictional objects, is the universe of some model[2].

Taken together, conditions (B)-(D) say that: (i) a 1-place logical predicate *C* of level 1 is defined by a single extensional function f_C , such that f_C is defined over all models for the language, and for every model **A** with a universe *A*, f_C(**A**) is a subset of *A;* (ii) the meaning of *C* is fully and precisely captured by f_C.[3]

Can 'unicorn' be construed in L as a logical constant in accordance with (B)-(D)? Strictly speaking, no. 'Unicorn' (as it is used in English) is a zoological predicate, and as such it is inapplicable to non-zoological, or at least to non-biological, objects – numbers, thoughts, planets, etc. It follows that the meaning of 'unicorn' would not be

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[2] In conversation Gómez-Torrente said that he took my definition to require that universes contain only existent objects. Although this clarification may suffice to remove the misunderstanding, I think an explanation of why the criticism does not hold is required to forestall further misunderstandings.

[3] This point is emphasized in Sher (1991), p. 56: "Condition (B) ensures that logical terms are *rigid*. Each logical term has a *pre-fixed* meaning in the metalanguage. This meaning is unchangeable and is completely exhausted by its semantic definition [i.e., the function defining it]. [Underline added]

accurately captured by a function defined over models with universes consisting of (non-empty) sets of such objects. Hence construing 'unicorn' as a logical constant would violate condition (D).

Suppose, however, that we relax (B)-(D) by allowing certain conventional extensions of partial functions capturing the meaning of logical constants. Suppose we adopt the following principles:

(a) If a function that *precisely captures* the meaning of a logical constant C is partial (i.e., defined over a proper subclass of all models), then a conventional extension of this function that does not *seriously* change the meaning of *C* is said to *capture** its meaning.

(b) One of the total functions that capture the meaning of *C* is (conventionally) designated as *precisely capturing** its meaning.

And we require that a logical constant be defined by a total function that *precisely captures** its meaning. Suppose, further, that the meaning of 'unicorn' is precisely captured by the partial function $f_{unicorn}$, where

(i) the domain of $f_{unicorn}$ is the class of all models whose universes contain real or fictional animals, and

(ii) for any model **A** in the domain of $f_{unicorn}$: if A is the universe of **A**, $f_{unicorn}(\mathbf{A})$ = the set of unicorns in A.

Then the *total* function $f^*_{unicorn}$, defined below is said to capture* the meaning of 'unicorn'.

For any model **A**,

$$f^*_{unicorn}(\mathbf{A}) = \begin{cases} f_{unicorn}(\mathbf{A}) & \text{if } \mathbf{A} \text{ is in the domain of } f_{unicorn}. \\ \text{The empty set} & \text{otherwise} \end{cases}$$

Let us further (conventionally) designate $f^*_{unicorn}$ as precisely capturing* the meaning of 'unicorn' in L. Then 'unicorn', as defined by $f^*_{unicorn}$, satisfies (B)-(D). Does it satisfy (E)?

Before turning to this question let me observe that if 'unicorn', thus defined, is considered a logical constant, Gómez-Torrente's criticism does *not* apply. The reason is that under this definition 'unicorn' has a non-empty extension in some models (namely, models whose universes contain mythological animals of the kind *unicorn*), and as a result '∀x~unicorn(x)' will not come out logically true. Putting this issue aside, let us proceed to condition (E). Does 'unicorn', under the above definition, satisfy condition (E)?

Condition E. Condition (E) says that a 1-place logical predicate of level 1, *C,* is defined by a function f_C which is invariant under isomorphisms of structures. That is, the following holds:

If **A** and **A'** are models with universes A and A' respectively, $b \in A$, $b' \in A'$, and the structures $\langle A, \langle b \rangle \rangle$ and $\langle A', \langle b' \rangle \rangle$ are isomorphic, then $b \in f_C(\mathbf{A})$ iff $b' \in f_C(\mathbf{A'})$.

It is clear that $f^*_{unicorn}$ does not satisfy this condition. Let **A** be a model whose universe, *A,* is a set of exactly one mythological animal – a unicorn, *b*. Let **A'** be a model whose universe, *A',* is a set if exactly one number, *b'*. Then the structures <*A*,<*b*>> and <*A'*,<*b'*>> are isomorphic, but $b \in f^*_{unicorn}$ (**A**) while *b'* Ï $f^*_{unicorn}$(**A'**). Similar considerations will apply to other construals of 'unicorn' as a logical constant of L: if the requirement that any set of objects (including sets of fictional animals) constitute the universe of some model for L is satisfied and the conditions (B)-(D) are satisfied, (E) will not be satisfied.

2. 'Heptahedron'. Like 'unicorn', 'heptahedron' , as it is used in L (a language whose English terms preserve their usual meaning), is neither a truth-functional connective nor, strictly speaking, a constant satisfying conditions (A)-(E): 'Heptahedron' is a geometrical predicate, and as such it is inapplicable to non-geometrical objects, e.g., thoughts; hence it is not defined over all models. Can we extend 'heptahedron' to all models along the lines described above, i.e., by a partly conventional function, $f^*_{heptahedron}$, that captures* it meaning? I will consider three cases:

(i) 'Heptahedron' in English is well-defined over one-sided surfaces in accordance with Hilbert & Cohn-Vossen's use (1952: 302-3), so that 'heptahedron' has a non-empty extension in some models. In that case we can construct $f^*_{heptahedron}$ in a way that captures* the meaning of 'heptahedron', e.g., by postulating that its value for all models over which 'heptahedron' is not defined is the empty set. But then the reconstructed 'heptahedron' does not satisfy (E). (The reason is similar to that described in (iii) below.)

(ii) 'Heptahedron' in English is restricted to two-sided surfaces in Euclidean space and we construct $f^*_{heptahedron}$ as assigning the empty set to *all* models. In that case, (the reconstructed) 'heptahedron' will not satisfy (B)-(D): (B)-(D) require that the definition of 'heptahedron' capture* its intended meaning in L, i.e., its English meaning. But if we define 'heptahedron' by $f^*_{heptahedron}$, this requirement is violated. Why? Because in that case the meaning of 'heptahedron' in L is the same as the meaning of '∅' in L or that of 'non-self-identical' in English. But 'heptahedron' is a geometrical concept, and as such its meaning in L (by assumption, its meaning in English) is *very different* from the meaning of the non-geometrical concept '∅' in L or that of 'non self identical' in English.

(iii) 'Heptahedron' in English is as in (ii), but we overcome the problem with (ii) by constructing $f^*_{heptahedron}$ as a function that "regards" one (conventionally designated) object with no geometric properties, *b'*, as a heptahedron in all models to which it belongs. In that case, however, (E) is violated. Why? Suppose **A** is a model with a universe *A* such that $f^*_{heptahedron}$(**A**) is the empty set. (There is such a model.) Take a model **A'** with a universe *A*' such that (a) *b'* is a member of *A'*, and (b) the cardinality of *A*' is the same as the cardinality of *A*. Let *b* be a member of *A*. Then the structures <*A*,<*b*>> and <*A'*,<*b'* >> are isomorphic, but *b* Ï $f^*_{heptahedron}$(**A**) while $b' \in f^*_{heptahedron}$(**A'**).

3. 'Male widow'. For similar reasons 'male widow' is not a logical constant of L.

It goes without saying that if we decide to use 'unicorn', 'heptahedron', and 'male widow' in a non-standard way (relative to their usual English use), then, depending on how we use them, they will or will not satisfy (LT). For example, if we decide to use them as "the empty predicate", i.e., as a predicate whose meaning is *fully captured* by a function that assigns to all models the empty set, they will. But in that case Gómez-Torrente's criticism will no longer hold, since under this use '∀x~unicorn(x)', '∀x~heptahedron(x)' and '∀x~male widow (x)' *will be* logically true.

This brings us to our next topic: the task (point, goal) of characterizations of logical constants.

4. Three Characterization Tasks

We may distinguish three types of precise and informative characterization of logical constants, according to the tasks they set out to accomplish: (i) a characterization of logical operators, (ii) a characterization of logical constants in logical languages, (iii) a characterization of logical constants in English (or some other natural language).

(i) *A Characterization of Logical Operators.* Logical operators, unlike logical constants, are not linguistic entities. Sometimes, however, the characterization of logical operators is given in terms of logical constants. A classical example is the characterization of the logical connectives as truth-functional. Although this characterization is formulated as a characterization of linguistic entities (logical connectives), its main point is not linguistic: Every Boolean truth-function is a logical operator, and the logical operators of sentential logic are exhausted by the Boolean truth functions. The challenge of developing an equally comprehensive, precise and informative characterization of logical operators for predicate logic was taken up by Mostowski (1957), Tarski (1966), Lindström (1966) and others, sometimes directly – as in Tarski (1966)[4] and Sher (1991, Ch. 4) – and sometimes through a definition of logical constants – as in Sher (1991, Ch. 3)[5].

Direct characterizations of logical operators – e.g., Tarski's characterization – are, to begin with, immune to criticisms of the kind made by Gómez-Torrente; characterizations of logical constants whose main aim (or one of whose main aims) is to characterize logical operators – e.g., my characterization – are, to begin with, partially immune. (The argument of the last section was intended to show that my characterization eschews this criticism altogether.) The importance of characterizations of logical constants partly lies in their embedded characterization of logical operators.

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[4] Although Tarski (1966) called the objects he characterized "logical notions", they were not linguistic (or conceptual) entities: "I use the term 'notion' ... to mean, roughly speaking, objects of all possible types in some hierarchy of types like that in *Principia Mathematica*. Thus notions included individuals ..., classes of individuals, relations of individuals, classes of classes of individuals, and so on." (P. 147) Such objects can be easily reconstrued as operators (here, functions from models to structures of objects within them).

[5] In Ch. 4 of Sher (1991) I give a characterization of logical constants "from below" that includes a direct characterization of logical operators; in Ch. 3 of Sher (1991) I give a characterization of logical constants "from above" that includes an indirect characterization of logical operators. The characterization I discuss in this paper is that of Ch. 3.

(ii) *A Characterization of Logical Constants in Logical Languages*. This is the direct aim of my characterization. The idea is that one way to describe the scope and nature of logic is to describe (or give a set of instructions for) the construction of a logical system, and one important step in such a construction is the construction of logical constants. My characterization of logical constants is of this kind. It says that if you start with a background language in which the distinction between logical and non-logical constants is not demarcated and you set out to construct a logical system with logical constants taken from this language, you proceed as follows: Either (i) you pick out a term that can be introduced as a logical constant in accordance with (LT) without any substantial change to its original meaning, and introduce it as a logical constant in the system you are constructing, or (ii) you pick out a term that cannot serve as a logical constant in accordance with (LT) as it stands, and change its original meaning so that it can. It is in these ways that you can introduce English terms like 'not', 'every', 'nine', 'the number of planets', 'unicorn', 'heptahedron', and 'male widow' (or symbols representing them) as logical constants in a logical system. 'Not' and 'every' require no substantive change; 'nine' can be used as a logical constant if it is construed as a 2nd-level predicate (predicate of predicates of individuals); 'the number of planets' can be used as a logical constant if it is construed as synonymous to (the 2nd-level) 'nine'; and 'unicorn', 'heptahedron' and 'male widow' can be used as logical constants if they are construed as synonymous with the empty predicate. To the extent that this characterization of logical constants concerns their use in *formal systems* and is not committed to preserving their meaning in a given background language, it is also immune to the type of criticism exemplified by Gómez-Torrente.

(iii) *A Characterization of Logical Constants in Natural Languages*. This is largely an empirical enterprise, the enterprise of determining what terms speakers actually use as logical constants, either commonly or in certain contexts or circumstances. It is probably true that most people normally use 'unicorn', 'heptahedron' and 'male widow' as non-logical constants, but this is an issue that neither Tarski nor I have much to say about.

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