

# The Barcan Formula in Metaphysics\*

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ABSTRACT: The Barcan formula (BF) is widely considered a threat to actualism. I show how BF can be cleared of such a charge by construing it as a bridge principle connecting modality *de dicto* and modality *de re* while retaining a Russellian robust sense of reality in modal matters.

Keywords: Barcan formula; modality; *de dicto* and *de re*.

RESUMEN: La fórmula Barcan (FB) se considera por lo general una amenaza al realismo. Muestro de qué modo FB puede verse libre de esta imputación si se construye como un principio puente que conecte la modalidad *de dicto* y la modalidad *de re* al mismo tiempo que retiene un sentido russelliano robusto de realidad en cuestiones modales.

Palabras clave: fórmula Barcan; modalidad; *de dicto* y *de re*.

## 1. Introduction

The Barcan formula is a schema introduced by Ruth Barcan Marcus as an axiom schema in her pioneering work on quantified modal logic (QML):

$$(BF) \ \Diamond \exists x \phi x \rightarrow \exists x \Diamond \phi x.^1$$

It is paraphrased as the schematic conditional that if it is possible that there be a  $\phi$ , then something or other is possibly a  $\phi$ . Together with its converse CBF it gives expression to the most straightforward way of combining modal operators with classical quantification. But it is customary nowadays to think of BF as posing a threat to a view in modal metaphysics known as ‘actualism’, roughly the claim that there are no non-actual (or ‘merely possible’) things.<sup>2</sup> This, we are told, is regrettable if true to the extent that BF is validated by the most straightforward systems of quantified modal logic and actualism is highly plausible and attractive in its own right. So it would be a

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<sup>1</sup> A version of it first appears as axiom schema 11 in Barcan (1946). Another formulation is

$$\forall x \Box \phi x \rightarrow \Box \forall x \phi x,$$

but the  $\Diamond \exists$  construal is more intuitively accessible.

<sup>2</sup> I will focus throughout on metaphysical modality and will have nothing to say about other sorts: epistemic, deontic, etc.



shame if BF had these anti-actualist (or ‘possibilist’) implications. Thus we read in Linsky and Zalta (1994, 431):

[S]imple QML has many controversial features, not the least of which are that it validates the Barcan formula and appears to require quantification over possibilities. Whereas possibilists employ distinctions that render these features of simple QML unobjectionable, actualists find the distinctions and the controversial features difficult to accept.

And in Williamson (1998, 258-259) we read:

[I]f possibilities are what could exist, then the anti-actualist can defend BF and BFC by positing non-existent possibilities and quantifying over existent and non-existent possibilities. Given actualism, there are no non-existent possibilities to be quantified over, and BF and BFC look much less attractive.

Much effort and ingenuity have gone into attempts to reconcile systems of QML that validate BF with a commitment to actualism. We will return to such efforts below. It is scarcely remembered, however, that the originator of BF had strong actualist leanings. These can be seen as closely allied with her endorsement of the principle. Here are representative passages from Marcus (1986):

In my own sketch of a semantics for modal logic [in Marcus (1962)], the domains of individuals assigned to alternative worlds were *coextensive*. Given that one of the worlds is the actual world, no entities are spawned that are not in this world and no entities of this world are absent in others. That is consistent with the axiom, variations on which came to be known as the ‘Barcan formula’. (195)

Later, following a brief sketch of Kripke’s variable domains semantics, we read:

Despite the elegant generality of the formal extension of model theory to modalities that allows variable domains, many of the examples given of possibilities are among those that could be accommodated by the theory of descriptions and do not seem to require variable domains. Nevertheless, the semantics *accommodates* possibilities. But modalities in their primary use concern counterfactuals about actual objects, and to reintroduce possibilities is to run counter to the admonition of Russell that we ‘retain our robust sense of reality’. (197)

See also Russell (1919, 152). A plausible reconstruction of the position here is as follows. Modality is primarily about actual things and their possible configurations. This allots a certain priority to so-called possibility *de re*, to what is possible for particular things, over what is possible in general, or possibility *de dicto*. Given this as a metaphysical default, a formal semantics of a particular sort that validates BF is called for. Formulas reporting certain *de dicto* modal facts, together with suitable instances of BF, would entail ones reporting certain *de re* modal facts. That modality *de dicto* of the relevant kind should have such *de re* modal implications provides some expression to the overall idea that possibilities *de dicto* do not float free from what is possible for particular things, thus allowing us to retain a robust sense of reality in modal matters.

My aim in what follows is to fill in the details of this initial reconstruction and make a case, given a general antecedent commitment to the Russellian sentiment, for BF as a metaphysical principle. This will require first and foremost delving into the distinction between modality *de re* and *de dicto*, which will be the first order of business.

My concern will be with BF and not with its converse CBF—it is only the former that is relevant to the question whether there are merely possible things.<sup>3</sup>

One last prefatory remark before turning to the details: It might be thought that a schema such as BF formulated with respect to the language of QML, being a logical principle, could not be understood as a metaphysical principle. Logic and metaphysics, however their respective aims are to be characterized, are distinct enterprises, and to consider BF as a metaphysical principle is to fail to heed this distinction. This charge runs deep and I cannot do it full justice here. For present purposes suffice it to say that despite being a schema for syntactic constructions of the language of QML, BF taken as a metaphysical principle should seem no more problematic than that a principle formulated in English should be understood as a metaphysical principle. As far as the relation between logic and metaphysics goes, the position I set out to defend would align itself in a fuller treatment of the topic with the following influential passage from Russell (1919, 152):

Logic, I should maintain, must no more admit a unicorn than zoology can; for logic is concerned with the real world just as truly as zoology, though with its more abstract and general features.

BF as a metaphysical principle would accordingly limit abstract and general modal features of reality. The Russellian orientation that considers BF in this way certainly raises important issues concerning the relation between the principle itself, with its assumed generality, and the specific truths that exhibit it, but in what follows I will dodge this larger issue and move back and forth between speaking of the obtaining of the principle in *re* and the truth of its particular manifestations.

## 2. De re *and* de dicto

Consider the following report:

- (1) Something might have occupied the corner.

This has a reading on which

- (2) it is possible for something in particular to occupy the corner,

and a reading on which

- (3) it is possible that there be something occupying the corner.

The facts reported in (2) and (3) are intuitively different: (2) reports what is possible *for* something, namely, occupying the corner (treating the property as monadic rather than dyadic for convenience), whereas (3) reports what is generally possible, namely, that there be something or other occupying the corner. In (2) the modal appears to modify a thing (*res*) bearing a property, whereas in (3) the modal appears to modify a statement (*dictum*).

We note in passing that this way of capturing the contrast between *de re* and *de dicto* is *prima facie* not via matters of scope, as in

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<sup>3</sup> Left out of consideration, then, are CBF's implications for the position known as 'serious actualism'—the view that in possible circumstances in which something does not exist there are no facts about it of any kind, including its very non-existence.

(2') for some  $x$  (possibly ( $x$  occupies the corner)),

and

(3') possibly (for some  $x$  ( $x$  occupies the corner)),

but, rather, via a difference between the dyadic possibility-for and the monadic possibility-that, which is how the tradition following Peter Abelard would have it.<sup>4</sup> The scopal contrast exhibited by (2') and (3') is a logical capture of an old metaphysical contrast.

Now, the status of generalities in *re* can seem perplexing, and *de dicto* facts of the form  $\diamond\exists x\phi x$  are generalities: they obtain to the extent that it is possible as a general matter that there be a  $\phi$ . It is *prima facie* plausible that what is generally possible should be constrained by what is specifically possible for particular things. This is so because it seems plausible that general possibilities obtain only because specific possibilities for particular things do. The alternative can easily strike us as mysterious when considered in light of certain analogies. Consider an example adapted from Russell (1919): say I met a man. My having met a man took place only because I met some man in particular. It is not as if I could have met a man as a general matter without having met some man in particular. We might ask whether or not the very fact that I met a man could have obtained had I not met Jones in particular but rather Smith instead. But we cannot take seriously the idea that I could have met a man as a general matter without having met some man in particular. Meeting a man does not float free from meeting a particular man. Assuming that meetings are parts of reality, what would it mean for meetings to obtain as a general matter without having witnesses in specific meetings among particular individuals?

Assuming that general possibilities of the form  $\diamond\exists x\phi x$  are also parts of reality, a way of capturing the idea that general possibilities obtain only because possibilities for particular things do is to commit to instances of BF as metaphysical truths. This would offer some vindication of the view that there is no deep metaphysical chasm between *de dicto* and *de re* modality. *De dicto* modality of a certain sort would then have *de re* modal implications: what things might be like in general would entail what is possible for particular things.

I regard such considerations as providing the most compelling support for BF as a metaphysical principle. I will return to them in more detail below after examining two competing defenses of BF as a metaphysical principle: a logic-first approach that issues from considerations concerning the logical status of the schema, and a metaphysics-first approach that issues from considerations concerning the nature of the totality of things. After detailing shortcomings of the other two general approaches to the topic, I will return to my preferred metaphysics-first approach that issues from considerations concerning the nature of metaphysical modality itself.

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<sup>4</sup> According to Abelard, genuine modality is really only of the former dyadic kind: modality as a mode of predication, modifying the subject-predicate—or better, the object-attribute—link. See Kneale (1962), especially the discussion at pp. 624–627. See also 2.5 of King (2004). I explore this and related matters in Chapter 1 of Simchen (2012).

### 3. *First strategy: logic-first*

One might argue for BF as a metaphysical principle indirectly via its theoremhood in systems of QML that have certain obvious theoretical virtues.<sup>5</sup> The naturalness, simplicity, and facility with metalogical results of the version of QML at issue might be seen as recommending instances of BF as metaphysical truths. An immediate problem with this line, however, is that it is far from clear what theoretical virtues of a formal system—straightforwardness of proofs of soundness and completeness, say—have to do with metaphysical truth. It can easily seem mysterious that the right logic metaphysically speaking should also be the one with the most elegant metalogic. A defense is surely needed for this type of consideration of metaphysical truth via theoremhood so as to dispel the air of mystery. And the explanatory burden here seems very high.

A more elaborate logic-first defense of BF is found in Williamson (1998) and (forthcoming a). A line of thought emanating from Kripke (1963) considers truth conditions for the schemata  $\Diamond\exists x\phi x$  and  $\exists x\Diamond\phi x$  without prejudice to what individuals there might be, and claims them to diverge. The first schema is made true by an assignment to  $\phi$  of an extension at some world that includes an individual in that world. The second schema is made true by an assignment to  $\phi$  of an extension at some world that includes an actual individual. Clearly, without prejudice to what individuals there might be, fulfillment of the first condition does not entail fulfillment of the second condition. The invalidity of BF is easily shown. Kripke's proposed variable domains semantics for QML assigns domains of quantification to worlds and lets the extensions of predicates at worlds be drawn from the union set of these domains. A foe of BF can take inspiration from such a framework and claim that while it is indeed possible that there be a talking donkey (let us suppose), it is not possible for any actual thing in particular to be such. What makes it generally possible that there be a talking donkey is not what is possible for anything actual. The possibility of there being a talking donkey is found compatible with there being nothing actual that is possibly a talking donkey, thus falsifying the relevant instance of BF.

Williamson's response to such claims is to point out that putative counterexamples to BF, couched in a variable domains semantics for QML, are offered in a metalanguage whose quantifiers do not abide by strictures specified for object-level quantification. Thus, a counterexample to BF requires an assignment of a non-empty extension to  $\phi$  at some world where this extension includes some non-actual individual and no actual individual falls in the extension of  $\phi$  at any world. The quantifiers of the non-modal metalanguage—e.g. 'some non-actual individual'—are presumed to range over individuals outside the actual domain of quantification. This makes purported counterexamples to BF invariably philosophically unsatisfying. Counterexamples to BF are undermined by the requirement that the non-modal metalanguage quantifiers function differently from object language quantifiers at the actual world.

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<sup>5</sup> Examples of such systems are SQML endorsed by Linsky and Zalta and LPC=S5 endorsed by Williamson. See Linsky and Zalta (1994) and Williamson (1998), respectively.

Let us observe that as a general matter discrepancy between assigned domains for meta- and object-level quantification are routine. To see why this is supposed to be problematic in the present case requires that we move beyond the viability of this or that model theory for QML. A variable domains approach clearly invalidates BF, but this is beside the present philosophical point. Williamson's thought, rather, is that under the *intended* interpretation of the non-modal metalanguage, the domain over which the metalanguage quantifiers range is restricted to actuality, which is also the domain of quantification that the intended interpretation of the modal object language assigns to the actual world. But because purported counterexamples to BF require that quantification in the metalanguage extend beyond that, there can be no genuine counterexamples to BF under the intended interpretation of the non-modal metalanguage.<sup>6</sup>

Williamson's complaint against purported counterexamples to BF, couched in a variable domain semantics for QML, raises doubts, however. The idea that under the intended interpretation of the non-modal metalanguage the domain of quantification is restricted to the domain of quantification of modal object language quantification at the actual world under the intended interpretation of the latter is not obvious. Consider, for example, the fact that quantification over possible worlds under the intended interpretation of the non-modal metalanguage is quantification over items that *prima facie* fall outside the range of modal object language quantifiers at the actual world under the intended interpretation of the latter. Observing strictures of modal object language quantification at the actual world in the non-modal metalanguage would seem to preclude quantification over possible worlds other than the actual world. Suppose, however, that in a reductionist spirit possible worlds are identified with maximally consistent sets of sentences while possible individuals are identified with sets of one-place open formulas. It is unclear why a foe of BF could not then hold that under the intended interpretation of the non-modal metalanguage the domain of quantification includes sets of formulas, whereas under the intended interpretation of the modal object language the actual domain does not.

It may also be wondered why counterexamples to BF couched in a variable domains semantics for QML could not in principle be given without the appearance of quantification over mere possibilia altogether. Setting the previous point aside, a foe of BF might say that the possibility that there be a talking donkey requires that the property of being a talking donkey be instantiated at some world or other, which obtains (let us suppose) for reasons having to do with the non-contradictoriness of the property, while the possibility for something or other of being a talking donkey requires that being a talking donkey be instantiated at some world or other by some ac-

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<sup>6</sup> The option of resorting to a modal metalanguage—as in, for example, Fine (1977)—is unsatisfying in a different way. Falling back on a modal metalanguage can leave us none the wiser about the status of BF. A foe of BF might try to argue in a modal metalanguage that the object language  $\Diamond \exists x \phi x$  is true just in case a world-proposition entailing that something  $\phi$ s might have been true whereas  $\exists x \Diamond \phi x$  is true just in case for some actual thing a world-proposition entailing that thing  $\phi$ -ing might have been true, and that fulfillment of the first condition does not entail fulfillment of the second. But our judgment as to whether or not this is really so does not seem robust enough to afford independent assessment of BF. See also the discussion in Williamson (forthcoming a, 13-14).

tual thing, which fails to obtain (let us suppose) for reasons having to do with the fact that none of the actual donkeys could talk and nothing that is not already a donkey could be a donkey. And it might be supposed that the variable domains semantics provides a formal framework for rendering such divergences perspicuous. The semantics portrays the possible instantiation of  $\phi$ -ing not by anything actual as the instantiation of  $\phi$ -ing by something non-actual, but the latter is merely a *façon de parler* for heuristic purposes.<sup>7</sup> This line of criticism of BF does not obviously require quantification over non-actual things. Talk of something non-actual  $\phi$ -ing is replaced by talk of the instantiation of  $\phi$ -ing being non-contradictory while every possibility of an actual thing  $\phi$ -ing is excluded. *If*, against appearances to the contrary, this type of formulation of counterexamples to BF requires illicit quantification over non-actual things after all, then this would need to be shown.

Even if we set all these issues aside, the point about comparing metalanguage quantification with object language quantification does little to unsettle the intuitive appeal of the purported counterexamples. As Williamson (1998, 264) observes,

[t]he foregoing remarks do not yet rehabilitate BF and [CBF], for they do not undermine the intuitive force of the supposed counterexamples. What they undermine is only the idea that the relativized domains approach constitutes a satisfactory theoretical underpinning for the supposed counterexamples.

We will return to the issue of why we find purported counterexamples to BF intuitively compelling in section 6 below. Clearly, an argument against an argument against the intended truth of an instance of BF does not gage the question of the status of BF as a metaphysical principle directly and leaves us hopeful for a more direct vindication of the principle.

A final stab at a logic-first vindication of BF (and CBF) broached by Williamson (forthcoming b) centers on the observation that a certain second-order comprehension principle validated by the second-order modal logic that naturally extends Kripke (1963) undermines the intended failure of first-order BF (and CBF). The principle is roughly that given any condition on individuals, there is a property that as a matter of necessity is instantiated by all and only the individuals satisfying the condition. The trouble is that it is a consequence of this comprehension principle that the property of not being a given individual (the individual's 'negative haecceity') exists necessarily. So with respect to any counterexample to BF offered under the auspices of Kripke (1963) there will be some individual that does not actually exist but whose negative haecceity does actually exist. But how, wonders Williamson, could such a property exist in a world in which the individual whose negative haecceity it is does not exist? Williamson then goes on to explore possible rejoinders on behalf of the foe of BF (and CBF) and finds them philosophically wanting. Unfortunately, neither the comprehension princi-

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<sup>7</sup> Such a position would constitute a resounding endorsement of a fundamental metaphysical divide between *de dicto* and *de re* possibility discussed earlier, which we have independent reasons to reject, but we can set this controversy aside for the purpose of examining the intrinsic merit of the response to the Kripke (1963)-inspired objection to BF. The present point is that Kripke's variable domain semantics might be viewed by the foe of BF as a useful heuristic and nothing more.

ple in question, nor the requirement that a negative haecceity existentially depends on the individual whose negative haecceity it is, is sufficiently intuitively compelling to provide an effective vindication of BF as a metaphysical principle, leaving us with the uneasy sense that the logical cart is being put before the metaphysical horse here.

Might BF be approached via reflection on the nature of the totality of things as such and on what it is for things to bear properties they do not in fact bear? We would be giving up the effort to argue indirectly via theoremhood, or via some self-defeating characteristic of purported testifiers to the invalidity of BF under the intended interpretation, and argue for BF as a metaphysical principle by considering what it is for things to possibly bear properties and whether or not there might have been more things than there actually are.

#### 4. *Second strategy: metaphysics-first—the totality of things*

An argument for BF as a metaphysical principle might be offered via independent considerations in favor of a modally invariant totality of things. A clear illustration of success for such a strategy is provided by mathematical cases, where the size and makeup of the realm of mathematical things is closed to modal variation. When it comes to mathematics, the plausibility of BF as a metaphysical principle is driven by considerations pertaining to the nature of mathematical entities and structure. As we will see, however, such considerations do not generalize easily outside the mathematical context. To appreciate this we turn to a case study brought to contemporary philosophical attention by Kripke (1980).

In the course of drawing the now-familiar distinction between the epistemic category of *a priori* truth and the metaphysical category of necessary truth, Kripke famously discusses Goldbach's conjecture as a prime example—the conjecture that every even number greater than 2 is the sum of two primes. It is a *conjecture*, Kripke reminds us, and so surely not an *a priori* truth. Suppose, however, that the conjecture is in fact true. Is it possibly false? Surely not, reasons Kripke. For if every even number greater than 2 is in fact the sum of two primes, then this fact about each such number is verifiable by direct computation. And so, assuming the results of direct computation to be necessary, it is necessary with respect to the number in question. How, then, could there have been an even number greater than 2 that is *not* the sum of two primes? Thus Kripke (1980, 36-37):

What would that mean? Such a number would have to be one of 4, 6, 8, 10, ...; and by hypothesis, since we are assuming Goldbach's conjecture to be true, each of these can be shown, again by direct computation, to be the sum of two primes. Goldbach's conjecture, then, cannot be contingently true or false; whatever truth-value it has belongs to it by necessity.

The argument is a tad compressed. We reconstruct it. Goldbach's conjecture is the claim that

(GC) for every  $x$  (if  $x$  is even and  $x > 2$ , then for some  $y, z$  ( $y$  is prime and  $z$  is prime and  $y + z = x$ )).

Possible falsity for GC is that

- (†) possibly (for some  $x$  ( $x$  is even and  $x > 2$  and for every  $y, z$  (if  $y$  is prime and  $z$  is prime, then  $y + z \neq x$ ))).

From this, Kripke reasons, it follows that

- (‡) for some  $x$  (possibly ( $x$  is even and  $x > 2$  and for every  $y, z$  (if  $y$  is prime and  $z$  is prime, then  $y + z \neq x$ ))).

But now, given the truth of GC, it is true for every even number greater than 2 that it is the sum of two primes. And this is necessary for each such number—it is verifiable by direct computation—which belies the existence of a number for which it is possible to be even, greater than two, and not the sum of two primes, as per (‡). For what could such existence amount to? It would render modal number theory  $\omega$ -inconsistent. We conclude, then, that if GC is true, then it is necessarily so.<sup>8</sup>

Let us pause to consider the grounds for the Kripkean transition from (†) to (‡). Why do we, following Kripke, find the transition from (†) to (‡) to be so obviously plausible? An initial thought might be that the transition is grounded in a modal logic that validates BF. The transition would then be a simple application of *modus ponens*. But, firstly, this cannot lie behind Kripke's own handling of the case for the obvious reason that Kripke's proposed version of QML famously invalidates BF. And secondly, and more importantly, to fall back on a certain logical framework merely pushes back the explanatory challenge: What is it in the metaphysics that should impel us to adopt a framework that validates such a principle? Kripke himself appears to subscribe to the intended truth of the particular instance of BF we are considering, and there seem to be other such endorsements found in his work,<sup>9</sup> but we need to look deeper into the metaphysics of the situation for a clue to the transition from (†) to (‡).

It might be thought that a distinctly metaphysical source for the transition is that a general hostility to mere possibilities would inform a more specific hostility to merely possible numbers, the kind of thing required to sustain the truth of (†) in the face of the falsity of (‡). For it seems that the only way for (†) to be true while (‡) is false is that there be merely possible numbers. If (‡) is false, then for none of the numbers in actuality is it possible to be even, greater than 2, and not the sum of two primes. Under such conditions, what might still render (†) true? Given the falsity of (‡), it will not be whatever is possible for any of the actual numbers. Nor, on the other hand, will it be whatever is possible for any other actual thing if we assume that for no non-

<sup>8</sup> Letting GC be  $\forall x(\text{even } x \wedge x > 2 \rightarrow \exists y \exists z (\text{prime } y \wedge \text{prime } z \wedge y + z = x))$  we have the following

Claim:  $GC \rightarrow \Box GC$

Proof: We suppose for *reductio* that GC while  $\Diamond \neg GC$  and let  $G(x)$  be:  $\text{even } x \wedge x > 2 \rightarrow \exists y \exists z (\text{prime } y \wedge \text{prime } z \wedge y + z = x)$ . If GC, then by UI for every  $n$ ,  $G(n)$ , which can be shown by direct computation; thus, for every  $n$ ,  $\Box G(n)$ . On the other hand,  $\Diamond \neg GC$  is equivalent to (†)  $\Diamond \exists x \neg G(x)$ , from which it follows that (‡)  $\exists x \Diamond \neg G(x)$ , which is equivalent to (‡)  $\exists x \neg \Box G(x)$ , which is  $\omega$ -inconsistent with  $\Box G(n)$  for every  $n$ .

<sup>9</sup> See 2.3 of Simchen (2012) for a reading of the cryptic unicorn remarks in Kripke (1980) according to which Kripke is tacitly committed to the idea that if possibly (for some  $x$  ( $x$  is the unicorn species)), then for some  $x$  (possibly ( $x$  is the unicorn species)).

number is it possible to be even—never mind being greater than 2 and not the sum of two primes. (Pre-theoretically, such possibilities as being even are tailor-made for numbers.) The only remaining plausible option, it seems, is that there be merely possible numbers.<sup>10</sup> And a general hostility to mere possibilia would rule this out, thus legitimating the transition in question.

However, it is unclear how to assess the question of the existence of merely possible numbers at such a level of abstraction. It is generally far from clear, for example, that a merely possible  $\psi$  is a  $\psi$  that is merely possible. Attributive adjectival constructions that do not license an inference from being a  $\phi\psi$  to being a  $\psi$  are amply familiar—a fake Rembrandt that is not a Rembrandt and a rubber duck that is not a duck are obvious examples. More relevantly, it certainly does *not* seem to be the case that a potential  $\psi$  is a  $\psi$ , one that is also potential. A potential parent, for example, is not a parent at all and so not a parent that is potential (whatever that might mean). Given such lack of clarity surrounding being a merely possible  $\psi$  in general, it seems far better to consider the specifics of the mathematical case and see whether they generalize outside the mathematical context, rather than consider the general case first and then see whether it applies to the specifics of the mathematical case.

##### 5. *The mathematical plenum vs. plena of all things*

A compelling metaphysical source for the Kripkean transition from (†) to (‡) is the nature of the structure of the numbers. We just mooted the idea that the only way for (†) to be true while (‡) is false is for there to be merely possible numbers. And yet, we now reflect, there cannot be additional numbers to those that actually exist: the structure of the numbers is fully saturated with its entities and is exactly the way it has to be. There is simply no room in the mathematical plenum, which is exactly how it has to be, to accommodate an additional number. Consider first whether there could have been an additional number occupying the location actually occupied by one of the actual numbers in its stead.<sup>11</sup> What makes this option impossible is that it is not possible that there be fewer numbers than there actually are due to the natures of the numbers, taken severally, which require each of the numbers to exist necessarily. We might then suppose that this by itself could carry the weight of a general denial of the existence of merely possible numbers. But it is not quite so. We would need to appeal to the nature of the mathematical plenum, the nature of the numbers taken collectively, to drive the general point home. For even if we assume that each of the numbers had to exist—thus ruling out the possibility of a merely possible number possibly occupying the location actually occupied by one of the actual numbers—this by itself does not yet force the conclusion that there could not be an additional number. For that result, the

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<sup>10</sup> We may suppose that on the present line of thought the Carnapian suggestion that the truth of (†) derives from the non-contradictoriness of the instantiation of being an even prime number greater than two and not the sum of two primes is not considered a plausible alternative.

<sup>11</sup> I set aside structuralist worries about how exactly a merely possible occupant of the location of one of the actual numbers is supposed to deliver the truth of (†) in light of the falsity of (‡).

essentialist point about the structure of the numbers made above—that it is fully saturated with its entities and is exactly how it has to be—needs to be appealed to as well. We might as well dispense with the independent appeal to the natures of the numbers taken severally. For when we said that the structure of the numbers is just as it has to be, this already included each of the numbers existing necessarily.<sup>12</sup> The present claim that there could not be merely possible numbers is in the first instance a claim about the numbers considered collectively rather than about each of the numbers taken severally.

Whether or not such considerations can be reasonably attributed, based on the meager textual evidence, to Kripke's own endorsement of the transition from (†) to (‡), they seem compelling in their own right. And the lesson to be drawn from this for possibilities for numbers is straightforward: If it is possible in general that there be a number answering a certain condition, then the condition is specifically possible for one of the numbers. If the condition could not possibly hold for any of the numbers, then it is not possible that there be a number answering the condition after all. (Conversely, and equally plausibly except not required for the transition from (†) to (‡), if a condition is specifically possible for a particular number, then it is also generally possible that there be a number answering the condition.) Abstracting further from the details of the case we can say that what is generally possible here—the possibility that there be so-and-so—is straightforwardly constrained by whether it is specifically possible for certain things to be so-and-so. The reason why we find this so plausible in the mathematical case is that the structure of the numbers is a plenum that is already the way it has to be.

Our next task is to assess whether this type of consideration could plausibly be extended outside the mathematical arena, thus providing a totality-of-things vindication of BF as a metaphysical principle. Consider the position defended by Linsky and Zalta, and, independently, Williamson, according to which if it is generally possible that there be so-and-so, then something or other in particular is possibly so-and-so. (See Linsky and Zalta (1994, 1996), and Williamson (1998, 2000).) The view in question is the metaphysical analog of a fixed domain semantics for QML, a point to which we will return. It is best illustrated by its handling of a familiar difficulty that arises for views that are committed to BF as a metaphysical principle.

Let it be the case that nothing  $\phi$ s and that  $\phi$ -ing is such that it cannot be had contingently, meaning that necessarily anything at all is either necessarily  $\phi$  or else necessarily not- $\phi$ . (Equivalently, it is necessary that anything that possibly  $\phi$ s necessarily  $\phi$ s.) Then under these conditions, and under an antecedent commitment to BF, the innocuous-seeming assumption that it is generally possible that there be a  $\phi$  yields a contradiction. For then, by the relevant instance of BF, something or other possibly  $\phi$ s, and thus necessarily  $\phi$ s, and thus actually  $\phi$ s, contradicting the initial assumption that nothing  $\phi$ s. And yet there is often considerable intuitive pressure to suppose, even under

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<sup>12</sup> The point is independent of any further structuralist insistence that the natures of the numbers themselves are exhausted by their locations in the mathematical plenum.

such conditions, that it is possible that there be a  $\phi$ . A representative example here is being Wittgenstein's child.<sup>13</sup> Wittgenstein had no children. Matters of tense aside, we are assuming in accord with BF that if it is possible that Wittgenstein should have had a child, then something or other is possibly Wittgenstein's child. And it is widely acknowledged that being Wittgenstein's child cannot be had contingently: it is necessary that anything at all is either necessarily Wittgenstein's child or else necessarily not his child.<sup>14</sup> (Equivalently, it is necessary that anything that is possibly Wittgenstein's child is necessarily Wittgenstein's child.) But then, the intuitive verdict that Wittgenstein might have had a child contradicts the plain truth that he had no children. For if it is possible that he should have had a child, then, we are assuming, something or other is possibly his child, and so, necessarily his child, and so, actually his child, contradicting his having no children.

The Linsky-Zalta-Williamson line addresses this problem by espousing the existence of a thing—a 'bare possibilium'—that might have been Wittgenstein's child, a thing that must be his child if concrete, and thus is his child if concrete. This thing, the bare possibilium, is not a concrete thing in actuality but might have been one. The claim that Wittgenstein has no children is true if read as the claim that nothing concrete is his child, and false if read as the claim that nothing is his child if concrete. The view has it that to *every* possibility that there be a  $\phi$  there corresponds something that is possibly a  $\phi$ . If nothing concrete can be found to possibly  $\phi$ , then the view has it that something nonconcrete is possibly  $\phi$ . There are as many things in actuality as are needed to sustain every general possibility of the form  $\diamond\exists x\phi x$ . There could not be more of them or fewer of them. Like numbers, things in general exist necessarily but not necessarily as concreta. This is the metaphysical correlate of a semantics that validates BF and CBF.

The view construes the claim that being Wittgenstein's child cannot be had contingently as the claim that it is being-Wittgenstein's-child-strictly-if-concrete that cannot be had contingently. 'Strictly-if' because the non-strict variant—being-Wittgenstein's-child-if-concrete—is had by anything that is not concrete, which for the view in question includes all the things that might have been concrete without being Wittgenstein's child. As a quick illustration of the point, consider the fact that I am actually brotherless. On the view in question a merely possible brother of mine, not actually concrete, might have been concrete, in which case it would have been my brother. However, if it were being-Wittgenstein's-child-if-concrete that could not be had contingently, then given that my merely possible brother has this property by dint of actually not being concrete, my merely possible brother would not only be my brother if it were concrete but would also be Wittgenstein's child if it were concrete—an unwanted result.

That things in general are like numbers in existing necessarily raises the question whether they are like numbers in other ways too, specifically in forming a structure

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<sup>13</sup> The example is from Williamson (1998).

<sup>14</sup> For present expository purposes we set aside the question of how to account for necessary predication without imputing necessary existence. See Chapter 3 and Appendix 1 of Simchen (2012) for discussion.

that is fully saturated with its entities. Earlier we witnessed the plausibility of the contention that if it is generally possible that some number or other satisfy a certain condition, then it is possible for some number or other to satisfy the condition. This was found plausible because the numbers form a plenum that is just the way it has to be. If the same could be said about things in general, then we would have found a clear totality-of-things vindication of BF as a metaphysical principle, whereby it would follow from it being generally possible that something or other be so-and-so that it be specifically possible for something or other to be so-and-so.

However, the considerations adduced above in favor of the Kripkean transition from (†) to (‡) seem not to apply beyond the mathematical case. We have no independent reasons for thinking that as a general matter the totality of all things, each existing necessarily, is a plenum that has to be the way it happens to be. Advocating a modally fixed totality of all things to match a fixed domain semantics for a system of QML that validates BF (and CBF) falls far short of providing substantive metaphysical grounds for supposing that there could not be things in addition to the things there are.<sup>15</sup> As we witnessed in the mathematical case, such a result will not simply follow from the necessary existence of things taken severally—an appeal to the realm of things taken collectively would have to be made. But here we clearly come up short. And the situation is made worse by the fact that the view under consideration is not meant to entail the necessitarian thesis that how things happen to be is how they must be. Given that room is to be left for contingency in the configuration of things, the task of showing that any such contingency is saturated with its entities seems all the more difficult. We have been given no substantive reason for thinking that as a general matter the realm of things is a plenum no matter how the things might have been configured.

#### 6. Third strategy: metaphysics-first—modality

We finally come to the most direct way of arguing for BF as a metaphysical principle that proceeds by way of considering metaphysical possibility as such and its determinants. This is the approach to *de dicto* possibilities and their relation to *de re* possibilities sketched at the beginning of the paper. Here we have another metaphysics-first approach whereby the truth of instances of BF is approached directly and not via the-oremhood, except that here it is not via consideration of the totality of things as such as in the previous strategy but of modality itself. The approach seems closest to Marcus's original motivations.

Letting *de dicto* possibility be what is generally possible without explicit regard to what is possible for particular things, on the present approach what is generally possi-

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<sup>15</sup> This is not to deny the systematicity in theoretical efforts to capture the idea, as exhibited by, for example, Zalta's Object Theory with its comprehension schema  $\exists x (A!x \wedge \forall F(xF \leftrightarrow \phi))$ —where *A!* is the abstractness predicate and *xF* is the encoding of property *F* by *x* (as opposed to the exemplification of the property by the thing)—and its ramifications. For a clear statement of the view, see Zalta (1993), and for a recent application to the philosophy of mathematics, see Linsky and Zalta (2006). The point in the text is rather that we have no independent grounds for espousing such principles.

ble is nevertheless determined by what is possible for particular things.<sup>16</sup> If something is generally possible, then it is so only because of a corresponding possibility for something in particular. As an illustration let us consider the question of what makes it the case that it is possible that there be a chair occupying an actually empty corner. The default intuitive answer is that what makes this generally possible is that something might have been a chair occupying the corner—presumably one of the actual chairs might have been such. And for that to be the case, presumably a counterfactual history ending with a chair in the corner would have had to diverge at some point from the actual history of the empty corner, a counterfactual history involving one of the actual chairs. The important point is that such general possibilities, or possibilities *de dicto*, have particular witnesses. A general possibility that there be a  $\phi$  does not float free of the possibility of  $\phi$ -ing for particular things. The monadic possibility-that obtains only because the dyadic possibility-for does. It is the latter that is prior in the order of metaphysical explanation. BF can be seen as giving expression to such priority.<sup>17</sup> We note that Barcan (1946) introduces the Barcan formula as a strict conditional: it is not possible for there to be a  $\phi$  without something possibly  $\phi$ -ing. This can be seen as giving an even closer approximation of the metaphysical priority under consideration. Perhaps an even closer approximation, employing further expressive resources, might be given by the subjunctive conditional that had it not been possible for anything to  $\phi$ , then it would not be possible that there be a  $\phi$ .

There remain the cases of purported general possibility for which there seem to be no plausible specific witnesses as discussed in the previous section. Purported troublemakers for BF have the following structure. It seems (A) possible that there be a  $\phi$ . And yet it also seems (B) impossible for anything to be a  $\phi$ , given that nothing is a  $\phi$ , and given that being a  $\phi$  cannot be had contingently. An anti-essentialist response is to proclaim that it is, after all, and against appearances to the contrary, possible for something that is not a  $\phi$  to be a  $\phi$ .<sup>18</sup> The plenulist response discussed in the previous section proclaims that it is possible for something contingently nonconcrete to be a  $\phi$ . Both anti-essentialism and plenulism respect Seeming (A) while denying Seeming (B), giving up on essentialist intuitions and on a Russellian robust sense of reality, re-

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<sup>16</sup> This can be easily extended to cover such mixed cases as the possibility that there be a talking donkey descending from Dapple the donkey, where the possibility is general on the position of being a talking donkey but particular on the second position of descending-from.

<sup>17</sup> What about general possibilities of the form  $\diamond\forall x\phi x$ ? Here the idea can be that such a possibility obtains only because it is possible for a plurality drawn from the totality of things to be such as to sustain the universality of  $\phi$ -ing per that plurality. This would still be an account of a monadic possibility-that in terms of a dyadic possibility-for, except now the latter holds with respect to pluralities. For present purposes we can set these complications aside.

<sup>18</sup> Thus Parsons (1995, 11):

Antiessentialism is required as follows. Certainly there might have been more porcupines than there are. So there must be another possible world in which there are things that are porcupines that are not porcupines in this world. [...] We thus need the possibility that a thing that is not a porcupine in this world is one in the other world; that thing thus cannot essentially be or not be a porcupine.

spectively. An under-explored alternative is to deny Seeming (A) while respecting Seeming (B). However, carving up an alternative position in logical space is clearly not enough. We must also provide a compelling diagnosis for why we are taken in by Seeming (A). An error theory of sorts is called for. Why does it seem generally possible that there be a  $\phi$  in the relevant cases where it is in fact not so? We consider the question through an example.

It can seem possible that there be an additional carbon atom to those that have appeared in the entire unfolding history of the universe. Here we find a general possibility with no apparent witness in a possibility for any particular thing. For what actual thing could have been a carbon atom without already being a carbon atom? Being a carbon atom, like being Wittgenstein's child, is not contingently instantiable: nothing could possibly have this feature without having it necessarily. Here is an endorsement of the idea for being gold and being human in Marcus (1971, 69):

Being gold or being a human being are not accidental. ... No metaphysical mysteries. Such essences are dispositional properties of a very special kind: if an object had such a property and ceased to have it, it would have ceased to exist or it would have changed into something else.<sup>19</sup>

So nothing is such that it might have been an additional carbon atom. We conclude that, initial appearances to the contrary notwithstanding, it is not really possible that there be an additional carbon atom after all. For suppose such a thing were possible. What would make it so? Recall the overall requirement to abide by Russell's robust sense of reality. This precludes an appeal to the vicissitudes of a merely possible carbon atom. There would have to be something about reality—robustly considered—that makes it so that it is possible that there be an additional carbon atom. Since what makes this the case will not turn (we are assuming) on how things might have gone with a merely possible carbon atom, and since it is not possible for anything that is not a carbon atom to be one, there is nothing about reality to sustain the general possibility that there be an additional carbon atom.<sup>20</sup> That said, plausibly it was possible at some point in the unfolding history of the universe that there be something (or things) to give rise to an additional carbon atom. This is not to say, however, that at any point it was possible that there be an additional carbon atom, which would require there to be something for which it was possible to be a carbon atom that was not already a carbon atom. Rather, it was possible that something (or things) give rise to an addi-

<sup>19</sup> Incidentally, given this characterization of being gold and being human as not contingently instantiable, the following somewhat tentative reconstruction of Marcus's position in Linsky and Zalta (1994, 436) is likely wrong:

From the fact that it is possible that  $b$  has a sister, BF requires that there exists something that is possibly  $b$ 's sister. Since  $b$  has no sisters, which existing object is it that is possibly  $b$ 's sister? Some actualists, notably Ruth Marcus [1986], might defend BF by pointing to an existing woman (possibly one closely related to  $b$ ) and suggesting that she is the thing which both exists and which is possibly  $b$ 's sister. But the great majority of actualists don't accept this idea, for they subscribe to certain essentialist views about the nature of objects.

<sup>20</sup> The Carnapian alternative that sublimates the general possibility by way of the non-contradictoriness of the instantiation of being a carbon atom distinct from any particular carbon atom surely does not abide by the Russellian admonition without a much-needed elaboration and defense. See 2.7 of Simchen (2012) for further discussion.

tional carbon atom. And for that to obtain, it was possible *for* something (or things) to give rise to an additional carbon atom.<sup>21</sup> This, again, does not require that at any point in the history of the universe there be something for which it was possible to be an additional carbon atom. It only requires that at some point in history there were potential propagators for such a thing. So it is not possible that there be an additional carbon atom after all, but our intuitive draw to the idea that this is possible can be explained in terms of the plausibility of the possibility that there be a propagator for an additional carbon atom. Accordingly, the intuitive appeal of such purported counterexamples to BF as that it is generally possible that there be an additional carbon atom without it being possible for anything in particular to be an additional carbon atom can be explained in terms of our mistaking the (genuine) general possibility that something propagate an additional carbon atom for the (merely purported) general possibility that something be an additional carbon atom.

### 7. Conclusion

Such is the most compelling case for BF as a metaphysical principle, a third defense that issues neither from the logical status of the schema nor from lofty considerations pertaining to the nature of the totality of things and what it is for them to possibly bear properties they do not in fact bear. Let me end by forestalling a natural misunderstanding of this third approach to BF as a metaphysical principle. It might be thought that the claim that it was possible that something give rise to an additional carbon atom is existentially committing to an additional carbon atom, raising anew the specter of mere possibilia. The thought is that from the general possibility that something propagate an additional carbon atom (letting  $\psi$  stand for the dyadic  $x$  propagating additional carbon atom  $y$ , the general possibility would be schematized as  $\Diamond\exists x\exists y \psi_{xy}$ ), it would follow by BF that something is such that it possibly propagates an additional carbon atom ( $\exists x\Diamond\exists y \psi_{xy}$ ), from which it would follow by EI that  $a$  possibly propagates an additional carbon atom ( $\Diamond\exists y \psi_{ay}$ ), from which it would follow by BF again that something is such that it is a possible outcome of propagator  $a$  propagating an additional carbon atom ( $\exists y \Diamond\psi_{ay}$ ). But the instantiation of the property of being a possible outcome of propagator  $a$  propagating an additional carbon atom is also the instantiation of the property of being a possible additional carbon atom. So we are back to the claim that something is possibly an additional carbon atom. And what might such a thing be? It appears that this could only be a merely possible additional carbon atom.

But ‘an additional carbon atom’ should not incur existential commitment in this way: the paraphrase in terms of existential quantification is spurious.<sup>22</sup> The point is widely accepted in cases such as

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<sup>21</sup> Here BF is followed with respect the possibility that something give rise to an additional carbon atom.

<sup>22</sup> There is some evidence that indefinites in English are never to be construed as existential quantifiers, but the general point need not concern us here. (See, however, chapter 4 of Steedman (2012) for a construal of indefinites as Skolem terms, taking inspiration from Fine (1995).)

(i) Bernard Ortcutt is a spy,

where there does not seem to be the requirement to construe this as

(ii) for some  $x$  ( $x$  spies and  $x =$  Bernard Ortcutt).

It would likewise be a mistake to construe

(iii) there is an additional carbon atom

as the *doubly* existential

(iv) for some  $x$  (for some  $y$  ( $y$  additional-carbon-atomizes and  $y = x$ )),

or to construe

(v) it is possible that there be an additional carbon atom

as

(vi) possibly (for some  $x$  (for some  $y$  ( $y$  additional-carbon-atomizes and  $y = x$ ))).

The correct renderings of (iii) and (v) are, rather,

(vii) for some  $x$  ( $x$  is an additional carbon atom)),

and

(viii) possibly (for some  $x$  ( $x$  is an additional carbon atom)),

respectively, with ‘an’ a determiner for ‘additional carbon atom’. In short, the intended reading of the claim that it was possible that something give rise to an additional carbon atom has ‘an additional carbon atom’ figure in characterizing the relevant monadic property of propagating an additional carbon atom, not unlike the way ‘a baby’ figures in speaking of fecundity as the ability to have a baby. BF can remain as attractive as ever without requiring us to abandon Russell’s robust sense of reality.

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