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Cognitive Profile in Learning Mathematics with Open Calculation Based on Numbers (ABN) Algorithm

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Abstract

The open calculation based on numbers (ABN) is an innovative mathematics teaching-learning methodology used with a huge number of school children. The aim of this work was to study the cognitive profiles associated to ABN method, compared to those following a closed procedure traditional methods based on ciphers (CBC). A total of 128 first-year students of Primary Education were evaluated on cognitive and mathematical performance. An experimental group (n = 74) and a control group (n = 54) were formed. The experimental group learned mathematic using the ABN methodology; the control group used a CBC methodology. The cognitive profile of the experimental group emphasized the significance of visuospatial working memory in mathematical performance. Students trained with ABN method seem to operate better with working memory, applying mentally visuospatial representations.

Keywords: ABN method, CBC method, mathematical cognition, working memory.

Resumen

En los últimos años un procedimiento metodológico innovador ha permitido plantear el aprendizaje de las matemáticas a partir de algoritmos abiertos basados en números (ABN) en una amplia población escolar. El objetivo de este trabajo ha sido estudiar los perfiles cognitivos asociados al método ABN, comparándolo con el alumnado que siguen un procedimiento de algoritmos cerrados basado en cifras (CBC). Se han evaluado componentes cognitivos y matemáticos a un total de 128 estudiantes de primer curso de Educación Primaria. Se distribuyeron en dos grupos, experimental (n = 74) y otro control (n = 54), que seguían un aprendizaje matemático con metodología ABN y CBC, respectivamente. El perfil cognitivo del grupo experimental enfatizó la importancia de la memoria de trabajo visoespacial en el desempeño matemático. El alumnado instruido con el método ABN parece operar mejor con la memoria de trabajo, aplicando mentalmente las representaciones visoespaciales en las que han sido entrenados.

Palabras clave: cognición matemática, memoria de trabajo, método ABN, método CBC.

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Introduction

The current position with respect to math performance has focused the researchers' attention for the study of learning difficulties in mathematics (Butterworth, Varma, & Laurillard, 2011). New intervention methods for improvement have been proposed (Aragon, Aguilar, Navarro, & Araujo, 2015; Martinez-Montero, & Sanchez, 2013). When we refer students who present difficulties in learning mathematics, mean that they are not able to appropriately deal when solving problems or calculations and his or her math skills do not match those that show other students of the same age (Fletcher, Lyon, Fuchs, & Barnes, 2007; Fuchs et al., 2008). A recent study confirmed that learning difficulties arise before formal education (Aunio, Heiskari, Van Luit, & Vuorio, 2015). The Science Education report by the European Commission (European Commission, 2015) warns of declining interest in the study of science and math in primary and secondary education and the need to increase demand for these studies on higher-level education, given the progress of the knowledge society.

For a long time in the school context, calculation learning and practice has been mechanically teach and supported in a traditional closed algorithms based on ciphers methodology (CBC). This practice should raise the development of lower order cognitive skills.

Some strong criticisms with CBC teaching method for basic math facts have been described (Kamii & Rummelsburg, 2008). It could undermine a significate student learning. This argument is based on the rejection of repetition and passive learning promoted by CBC learning. Students could not understand the underlying operations principles. Additionally, there is some empirical evidence that CBC methodology can be disadvantageous to learning mathematics. Martinez-Montero (2011) points out some: impoverishment of strategies and spontaneous student's methods to deal with tasks of non-routine calculation; nesting serious conceptual errors in calculation details hidden by the structure of the classical algorithms; finally, the lack of significance of the quantities expressed in ciphers in a broader range than expected age.

Similarly, Torbeyns, Verschaffel and Ghesquiere (2005) suggest that mathematics learning must go "beyond an expert routine; this is the ability to quickly and accurately solve mathematical tasks through standardized strategies" (p. 1). These authors state that students must develop expertise in calculating in a flexible way, creating and using significant strategies such as, for example, compensation strategies. They are essential for basic numerical facts. Moreover, the development of self-confidence in their own strategies.

There are several alternative procedures to address and solve mathematical learning problems arising from CBC methodology. One is called Open Algorithm Based on Numbers (ABN) teaching method. ABN is based on methodological principles developed by Martinez-Montero (2011), method's designer. The name ABN describes the main characteristics of open algorithms. These are: (a) Open Algorithms (A). There is no single way to solve them; each student can do this differently, depending on their rate of learning, math proficiency and calculation strategies. This new methodology allows each student to solve operations according to their capacity. This means that an effective motivation improvement and a satisfactory change in the attitude towards mathematics can be reached (Martinez-Montero, 2010; Martinez-Montero & Sanchez, 2011); (b) Based on Numbers (BN). It means that the algorithm always works with numbers. In any cases the complete numbers are combined with all its meaning, unlike traditional algorithms (CBC) that are based on ciphers, and it does not work with total quantities; (c) The ABN method is transparent. When children are solving any math task, no calculations or intermediate processes are hidden. The ABN method main target is that students understand the basic process of what they are doing all the time; (d) Math facts could be solved from left to right side or, if necessary, from right to left. The directions that students take in solving calculation tasks will be a function on their own learning style and rhythm. His or her own technique in applying mental calculation strategies in written format. The most commonly strategies used are decomposition, rounding and compensation.

ABN method has precedents some educational proposals launched in the Netherlands to renew the mathematics teaching and learning in general and particularly teaching methodology for calculation. This was called "realistic mathematics." This is oriented to development mathematical competence and fostering mathematical reasoning through manipulatives and stimulating instruments for students in order to increase motivation and attention (Van den Heuvel-Panhuizen, 2000). So far three paths teaching and learning have been implemented: first, for calculating with whole numbers in the lower grades of primary school (Treffers, Van den Heuvel-Panhuizen, & Buys, 1999), second for primary school higher grades (Van den Heuvel-Panhuizen, 2001), and third for measurement and geometry in primary school early grades (Vanden Heuvel-Panhuizen & Buys, 2005). These paths have been successful and have led to a thorough review of our thinking in mathematics teaching (Van den Heuvel-Panhuizen, 2008).

Implementation of ABN procedure begins in the academic course of 2008/2009 in a group 1st grade Primary Education in a public school at Cadiz (Spain). One year later, ABN extends in four more schools in the same province with approximate 125 students first graders. During the 2011 to 2012 different nationwide schools started implementing ABN method, distributed by more than ten Spanish regions. Throughout 2013 keep growing the Spanish public schools using this method, and starting from the first year of preprimary education. In addition ABN begins to expand internationally in other countries such as Mexico, Argentina or Chile. But there are still no published results on these experiences. According to data provided by Cantos (2016) currently between 6.000 and 7.000 classrooms follow the ABN methodology, representing an approximate total number of 200.000 students learning math with ABN.

Parallel to this procedure dissemination, some studies on ABN methodology have been developed for analyzing and comparing reached outcomes comparing to the CBC methodology. Results suggest a significant improvement for students instructed by ABN (Martinez-Montero, 2011).

Moreover, Bracho, Adamuz, Gallego, and Jimenez (2014) studied the development of the number sense reached at the end of the primary school second grade after using ABN methodology, paying special attention to the results obtained by students with different learning proficiency. The data show significant differences between mathematical competences achieved in the group using the methodology ABN compared to the control group.

The study by Bracho and Adamuz (2014) was investigated the results obtained by individual cases of students with specific educational needs. The main target was to analyze the number sense proficiency, paying particular attention to the different rates of learning existing in the classroom. Among the participants in the experimental group the results were considered effective, considering that some students had a limited intellectual capacity, or autism spectrum disorders.

Regarding the acquisition of numerical sense, it is necessary consider what cognitive processes are necessary to enable its development. The meta-analysis conducted by Peng, Namkung, Barnes and Sun (2016), after 110 studies, concludes that working memory has a strong relationship with problem solving and calculation, especially students with learning difficulties in mathematics. Rubinsten and Henik (2009) pointed out that the learning difficulties of mathematics should originated from a deficit in general domain cognitive abilities, such as working memory, the fluid intelligence or processing visuospatial (Aragon et al., 2015). Less conclusive are shifting and inhibition of irrelevant information skills (Bull & Lee, 2014). Consequently, a method which will provide a considerable gain in math skills and coping with the difficulties in this school tasks, should bring a cognitive benefit, or developing skills for contributing to an efficient general performance.

While ABN method started in our school context in 2008, contrasted studies linking this approach to cognitive performance parameters involved in mathematics learning at an early age are necessary. The confluence of two different methods of teaching mathematics in our school context, allows us to compare differences in cognitive profiles of students in each. In this sense, the main target of the study was to compare how cognitive processes operate in both ABN and CBC methods. Specifically, first, we studied the working memory, short-term verbal and visuospatial memory, and fluid intelligence, associated with the ABN method for teaching mathematics. Second, the ability of these cognitive processes have to rank students according to the type of instruction received is studied.

Method

Participants

This research involved a total of 128 first-year Primary Education students, whose ages ranged between 71 and 84 months (*M* = 77.87; *SD* = 3.24). 63 were boys (*M* = 77.73; *SD* = 3.16) and 65 girls (*M* = 78.0; *SD* = 3.34). Of all participants, 74 were instructed by ABN method, composing the experimental group, and 54 the control group. The control group received mathematical training program with the traditional CBC method since the beginning of the kindergarten. For the experimental group, 36 were boys (*M* = 76.75; *SD* = 3.15) and 38 girls (*M* = 76.82; *SD* = 3.5). Similarly, of the 54 control group students, 27 were boys (*M* = 79.04; *SD* = 2.7) and 27 girls (*M* = 79.67; *SD* = 2.27). Participants were assigned to 9 different classrooms. Five classrooms were taught math with ABN method and 4 classrooms with CBC method. These classrooms belonged to 4 different schools: two publics and two privates. The students of these schools come from middle and middle-low socioeconomic status. Sample was selected by a non-probabilistic procedure, since there was homogeneity of the population available for this type of study. Students who had special educational needs after a school psychologist's report did not participate in the study. No participant was excluded based on their results in mathematics assessment.

Instruments

Early Numeracy Test-Revised (ENT-R) (Van Luit et al., 2015). This test is designed for early math knowledge assessment. An ENT-R computerized and adapted into Spanish version, was used. The ENT-R consists of two subtests composed of tasks that assess mathematical competence. The first subtest called relational, included tasks on comparison, classification, one to one correspondence and serialization. The second subtest, called numerical included tasks on verbal counting, structured counting, counting (without pointing), general knowledge of numbers and estimation. The test has a total of 45 items, rated with a point when child reaches the correct response. The highest score is 45. ENT-R has three versions (A, B and C). All versions are psychometrically equivalent. ENT-R Cronbach's alpha was .92 (CR = .67; AVE = .47).

Automated Working Memory Assessment (AWMA) (Alloway, 2007). AWMA evaluates verbal working memory and visuospatial memory using simultaneous storage and processing information tasks. Tasks involving just the storage of information are used to measure visuospatial short-term verbal memory. AWMA provides three measures for short-term verbal memory, three measures for visuospatial short-term memory, three measures for verbal working memory, and three for visuospatial working memory. The whole test has of a total of 12 tasks and is aimed for people between 4 and 22 aged. An AWMA adapted into Spanish version was used (Injoque-Ricle, Calero, Alloway, & Burin, 2011). Although AWMA has a total of 12, in this study just four tasks were used, and described below:

- *Non-word recall*. In this task, the participant must pronounce a non-word sequence in the same order in which they have been heard. The task involves of 6 blocks with 6 items each. Items' amplitude is the same as the number of blocks. The main target is the evaluation of verbal short-term memory. Cronbach's alpha was .81.
- *Dot matrix.* In dot matrix, the child is asked to point to the squares of a 4-by-4 matrix where a sequence of red dots appeared in the same order that they were presented. The participant must indicate the matrix square in which the red dot appeared in the same order in which they were presented. Dot matrix task consisted of 9 blocks of matrix with 6 items each. The main target is the evaluation of visuospatial short-term memory. Cronbach's alpha was .91.
- *Backward digit recall*. Several sequences of numbers that increase in each test are verbally presented. The participant must remember and pronounce them in reverse order. This task was composed of six blocks of six items each. The main target is the assessment of verbal working memory. Cronbach's alpha was .89.
- *Odd-One-Out.* In the odd-one-out task, sets of three shapes in a three square matrix are shown on the computer screen, two are the same and the third one is different. The child is asked to indicate which one is the "odd one out" for each set. At the end, the child sees the matrix without any shapes and is asked to indicate where the odd shape had been in each set, in the order they had been presented. This task was composed of 7 blocks of six items each. The main target is the assessment of visuospatial working memory. Cronbach's alpha was .91.

Raven's Progressive Matrices Test (Raven, 2005). In order to evaluate the *g* factor (fluid intelligence), the classic Raven's Progressive Matrices (color version) test was used. With this test a measure of intelligence without cultural influence was obtained. The user is able to make sense of unstructured materials, developing logical relationships between a figure with some lacks in any part or feature. This can be completed by choosing one of several existing alternatives. Participants must use non-verbal ideas in order to solve the question and assimilate the pattern or structure presented. Cronbach's was .82 (CR = .74; AVE = .50).

Procedure

Before starting the study, the informed consent from students' parents was obtained. The school where research was conducted also provided informed consent. We were also held motivation meetings with school teachers. In this meetings research procedures were explained and, finally, a final results report for parents and school administrator was provided. The evaluation was performed early in the academic year 2013-2014, in classrooms with appropriate environments for test administration, free of noise and distractions, by trained psychologist. Each test was individually administered, during school hours, but seeing student's free time. An evaluation session between 15 and 30 minutes, per each test was used. This way of organizing the evaluation helped reduce participants' fatigue.

Both experimental and control groups participants were learning mathematical content either with ABN or CBC method, respectively, from the beginning of their schooling (3 years aged). ABN methodology involves teaching mathematics throughout schooling in preprimary and primary education as a consequence of a formal decision made by the school board. This decision comprises participating teacher's special training in ABN methodology and counseling by ABN experts. Teachers explain ABN specific content for each course, while respecting the general educative targets, content and competences required by the educational management system for each school year. Consequently, ABN method is not promptly given an experimental program in a particular classroom, but immersion in this type of methodology of teaching and learning as a strategic decision by the school board for all preprimary and primary education. The contents taught in preprimary and primary education by participants in the experimental group were based on three documentary sources: the work activities of the blog "ABN algorithms. For a simple, natural and fun math "(Martinez-Montero, 2008), the activities of the open distribution web "Actiludis' (VV.AA., 2012) and the textbook by Martinez-Montero and Sanchez (2011).

On the other hand, the CBC methodology refers to the common and traditional method of teaching mathematics to teach contents on preprimary and primary education. Students who follows the CBC method is majority in the Spanish educational system. The control group followed a teaching mathematics textbooks based on Spanish publishing (VV.AA., 2010, 2011, 2014a). They all teach mathematics with a CBC approaches adjusted to the contents required by the educational management system for Early Childhood Education.

In both cases, the standard required curriculum for each course was taught. The curriculum was instructed in the same teaching times planned by the Ministry of Education. The same number of hours and training sessions for the control and experimental groups were provided, given that the prescriptive for preprimary education level it is that there is not a specific time for each educative matter, given the inclusive status of this educational grade.

Data analysis

After sample's homogeneity calculating, Levene statistic test was significant at 95% level of probability for all cognitive assessments used. Then to carry out a study on the differences between the students' cognitive patterns trained in early mathematics competence between the traditional CBC and ABN methods, different statistical descriptive and inferential analysis with SPSS version 22 (U-Mann Whitney contrast and effect size tests) were calculated. A double statistical checking with a linear regression analysis and discriminating analysis were also achieved. No lost cases in the study were registered because data collected in this research was conducted in a single time.

Results

Regarding the descriptive analysis (Table 1), wide differences between control and experimental group used different tests were not detected. However, in order to verify whether the students trained cognitive functioning with either method varied, a stepwise regression analysis was calculated, using as predictor variables short-term memory and verbal working memory, visuospatial memory, and the fluid intelligence.

Table 1

Descriptive Statistics, U-Mann Whitney and Effect Size Result for the Experimental and Control Groups

	Experimental	Control		
	M (*DT*)	*M* (*DT*)	*p*	*d*
ENT-R	33 (5.74)	32.09 (4.03)	.429	.18
Nonword recall	119.98 (8.11)	113.56 (8.93)	.000	.75
Dot matrix	115.91 (15.06)	116.16 (12.62)	.703	-.01
Backward digit recall	105.66 (13.97)	111.63 (12.36)	.003	-.45
Odd-one-out	116.26 (13.65)	113 (15.10)	.160	.22
Raven	25.38 (3.9)	23.54 (4.3)	.024	.44

The results of linear stepwise regression analysis found cognitive profiles show clearly differentiated according to the type of instruction. First, the result obtained in the stepwise regression analysis performed for the control group (Table 2) is presented. This analysis was conducted in order to study the statistical weight that cognitive variables assessed showed in explaining the dependent variable: early math skills in the control group receiving CBC instruction.

Table 2

Results of Linear Stepwise Regression Analysis for Control Group

Model	R	R^2	R^2 corrected	Statisitic Standard error estimated	Change R2	Change F	Sig. Change F	Durbin Watson
1	.466a	.217	.202	3.50	.217	14.43	.000	
2	.539b	.290	.262	3.36	.073	5.24	.026	1.637

Note. a. Predictors variables: (Constant), Raven; b. Predictors variables: (Constant), Raven, Backward Digit Recall; c. Dependent Variable: Early Math Skills (ENT-R).

After a stepwise regression analysis, two models emerged. Last model offered a higher explanatory power. For the control group the multiple correlation coefficient was R = .539 and the determination coefficient R^2 = .290, that adjusted to R^2 = .26. Regarding the statistical model validity, the D Durbin-Watson was calculated obtaining D = 1.63. This result confirmed absence of negative self-correlation (values close to 4) and positive self-correlation (values near 0). The cognitive variable explaining further variation in the dependent variable (early mathematical ability) was fluid intelligence, assessed by Raven test (β = .410), then followed by verbal working memory, assessed by Backward Digit Recall task (β = .276).

On the other hand, also was studied the significance that cognitive variables had on the early math skills performance variation for students in the experimental group, instructed by mathematical ABN method (Table 3).

Table 3

Stepwise Linear Regression Analysis Results for the Experimental Group

Model	R	R^2	R^2 corrected	Estadísticos de cambio Standard error estimated	Change en R2	Change en F	Sig. Change F	Durbin Watson
1	.581a	.337	.328	4.70	.337	36.64	.000	
2	.645b	.416	.400	4.44	.079	9.63	.003	1.87
3	.682c	.465	.442	4.28	.049	6.40	.014	

Note. a. Predictors variables: (Constant), Raven; b. Predictors variables: (Constant), Raven, Non-word Recall; c. Predictors variables: (Constant), Raven, Non-word Recall, Odd-One-Out; d. Dependent Variable: Early Math Skills (ENT-R).

The stepwise regression analysis showed three models. The third model provided the highest explanatory value. The multiple correlation coefficient obtained was R = .682 with a determination coefficient R^2 = .465, and adjusted R^2 = .442. This model showed that the fluid intelligence variable had highest statistical weight on the dependent variable, assessed by Raven tests (β = .352), then followed by visuospatial working memory assessed by Odd-One-Out task (β = .284), and then short-term verbal memory assessed by non-word Recall task (β = .274). In order to verify the statistical model validity a *D* Durbin-Watson was calculated, obtained a value of *D* = 1.87 (a value close to 2.0 would be considered statistically valid).

Table 4

Discriminant Analysis Results for All the Variables Included in the Study

		Group predicted (*)		
		Experimental	Control	Total
Counting	Experimental	57	17	74
	Control	15	39	54
%	Experimental	77	23.0	100
	Control	27.8	72.2	100

Note. * 75% of original cases correctly classified.

In addition to regression analysis, a discriminant analysis was calculated looking for an arithmetic function that properly classifies participants based on variables evaluated data. The distribution achieved was compared with the real data, finding a classification matrix where the diagonal line represented the well-classified participants' percentage, and extradiagonals data represented the false positive and false negative individuals on the classification process (Table 4).

According to discriminant analysis data, 75% of participants were correctly classified within their own group. Consequently, it could assume that a pattern defining the students' performance in the variables analyzed, based on their instruction method emerged. Complementarily, an equal group's contrast by Wilks's *Lambda* statistic and determined by a *Chi-square* approximation was calculated. The result allowed to accept the differences between experimental and control groups (Wilks's *Lambda* = .747; X^2 = 35.92; p < .0001).

Discussion

In this study the statistical weight that several cognitive skills had in students' mathematics performing after the teaching mathematics methodology received in classroom was analyzed. Calculation is a good predictor of learning difficulties in mathematics. When students have problems in this skill during primary education, they preserve those difficulties during subsequent schooling (Geary, 2004). ABN methodology is characterized by generating a powerful mental arithmetic learning (Martinez-Montero, 2011). After the results of this study, a strong contribution of cognitive skills was found, when school children solved basic mathematic tasks, such as numerical activities used in the ENT-R test.

This work found that furthermore than a higher statistical weight for cognitive variables (such as working memory, short-term memory and fluid intelligence), on explaining early math skills for students who worked with ABN, they also exhibited a different cognitive profile compared to those using traditional teaching learning CBC approach. This cognitive profile shared common traits, such that the most explanatory variable was fluid intelligence for both, experimental and control groups. However, all other variables (working memory and short-term) changed both qualitatively and quantitatively.

For students taught with an ABN methodology, visuospatial working memory showed greater statistical weight, followed by short-term verbal memory. However, when teaching was through the CBC method, cognitive variable with higher statistical weight was verbal working memory, after fluid intelligence (Aragon, Navarro, Aguilar, & Cerda, 2015).

The explanation of these results might be because the teaching methods demands. They help to development of some cognitive domain-general skills. Those domain-general skills are launched when domain-specific skills, linked to mathematical performances, are necessary to implement.

ABN is a method in which students are trained with several manipulative and figurative resources (VVAA, 2014b). However, students learning in a traditional way, usually work with more abstract materials. So, mental calculation tasks in the CBC group involved that ciphers are retained in memory, and then be replaced in the appropriate order. But students instructed with ABN method seem operate more efficiently with working memory, mentally applying visuospatial representations that have been trained (Martinez-Montero, 2011).

According to the statement, students trained with a traditional teaching CBC procedure required a verbal working memory, in order to maintain and manipulate numbers in memory to reach the wanted result. On the other hand, the modus operandi of the students taught with ABN approach, mental calculation tasks would need a higher use of visuospatial working memory to manipulate the temporarily stored items into verbal short term memory, in order to solve it. Similarly, the difference in statistical weight that cognitive abilities have in explaining math skills suggest that students trained by ABN method are faster on calculation tasks. Those data are coincident with findings by Martinez-Montero (2011). Further memory skills development involves higher calculus fluency, and saving cognitive resources. Those savings would result in less timing consumption in solving mathematics tasks (Alloway, Gathercole, & Pickering, 2006; Alloway & Passolunghi, 2011; Bugden, Price, McLean, & Ansari, 2012; Passolunghi & Pazzaglia, 2005). Similarly, a cognitive overload should involve more likely to commit mistakes on the solving the math tasks. This should characterize students following the traditional teaching approach comparing to those who are taught by ABN methodology.

Some limitations of this study were the sample size, characteristic of the pilot studies, which proper reduces data generalization. Also, from a methodological point of view the study was limited by a non-probabilistic sample selection, and by quasi-experimental design, widely used in education research. However, the study proposes a research topic that will allow in the future tracking participants with a longitudinal design, in order to understand if cognitive profiles remain stable throughout schooling.

ABN method, as potential alternative method of learning-teaching mathematics generated some advantages. ABN contributed to develop those domain-general cognitive processes that are significant for success in mathematics, such as working memory. We expect that a pedagogical action performed from a scientifically well-founded method would improve student learning at risk for persistent mathematics difficulties in mathematics.

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